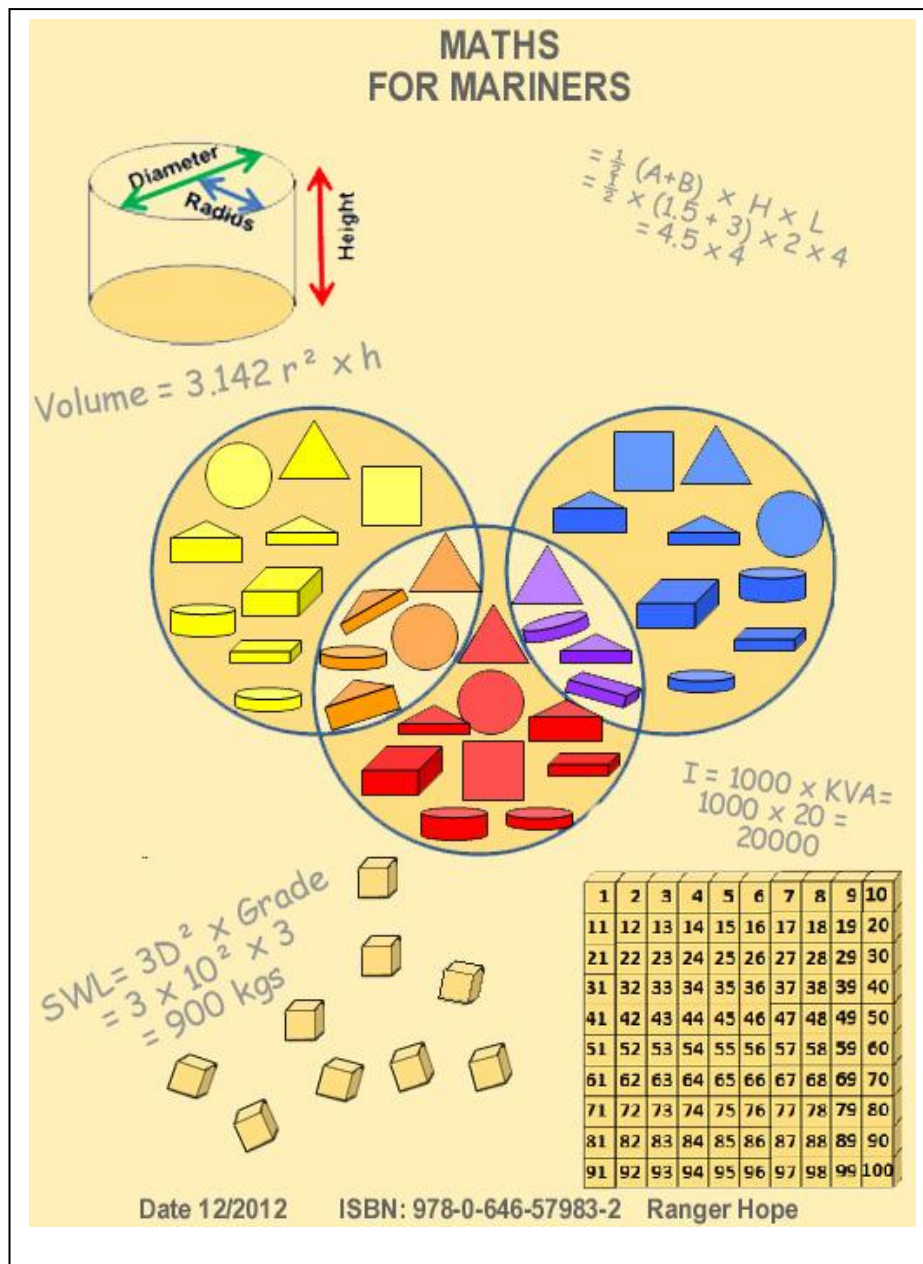


# MATHS FOR MARINERS



This text is provided for research and study only on the understanding that users exercise due care and do not neglect any precaution which may be required by the ordinary practice of seamen or current licensing legislation with respect to its use. No copying is permitted and no liability is accepted resulting from use.

*Ranger Hope © 2014*

# **Introduction**

## **Chapter 1: Logic**

- 1.1 Logic, attributes and sets
- 1.2 Number, value and counting
- 1.3 Numbering systems and number base
- 1.4 Scientific notation - exponents and roots

## **Chapter 2: Calculations**

- 2.1 Addition, subtraction, multiplication and division
- 2.2 Fractions and decimals
- 2.3 Calculating machines
- 2.4 Elementary algebra
- 2.5 Trigonometry

## **Chapter 3: Measuring**

- 3.1 International System of Units
- 3.2 Derived units and formulas
- 3.3 Measuring tools

## **Chapter 4: Shape, size and capacity of tanks**

- 4.1 Definitions
- 4.2 Areas of common shapes
- 4.3 Volumes of tanks
- 4.4 Pumps and flow rate capacity

## **Chapter 5: Calculating fuel requirements for a voyage**

- 5.1 Endurance
- 5.2 Specific fuel consumption and fuel coefficients
- 5.3 Propeller pitch

## **Chapter 6: Calculating a vessel's draught and trim**

- 6.1 Displacement and buoyancy
- 6.2 Moments and longitudinal trim
- 6.3 Moments and transverse trim

## **Chapter 7: Lifting equipment**

- 7.1 Definitions
- 7.2 Strength of rope, wire and chain
- 7.3 Levers and moments
- 7.4 Blocks, tackles and purchases
- 7.5 Friction and efficiency

## **Chapter 8: Structure and strength**

- 8.1 Stress and strain
- 8.2 Strength

## **Chapter 9: Refrigeration**

## **Chapter 10: Electrical calculations**

## Introduction

This text is intended for those studying to begin a maritime career. It aims to jog the memory of those who may have long since left school or assist others who's arithmetic, algebra, measure and geometry skills could be bettered.

This is a reference for formulas and methods required to solve mathematical problems common in the mariner's workplace. Although each section begins with basics, it is not intended to be read from start to finish. However, thumbing back to an earlier chapter will often be helpful in refreshing some underpinning concepts.

I learnt arithmetic at school entirely by the rote method of daily chanting times tables. But as I progressed I became entirely confounded by the mystery of slide rules, logarithms and calculus. I never really understood logic or numbers and like many I have survived with imperfect memories and a reliance on calculators. So this text is a personal journey of re-learning how to crawl before running. The first chapter on logic, sorting, base and numbering systems is provided for those who may also have been dozing at the back of the class.

# Chapter 1: Logic

## 1.1. Logic, attributes and sets

### Definitions

*Logic* refers to consistent rules where actions have predictable results.

*Geometry* describes how to draw shapes and defines the mathematical rules (relationships) concerning their construction.

*Mathematics* or *maths* describes the logical processing information using numbers.

*Algebra* enables logical solutions by balancing (equating) a set of calculations with a known value against one with a partially known value.

An *attribute* is that observable feature of a thing that makes it similar to or dissimilar from another thing. It defines what makes an individual.

To *sort* is to make collections (sets) of individuals determined by their common attributes.

A *set* is a group (collected) of individuals with one or more common attributes.

A *number* is a sequenced symbol used to represent an individual unit or the quantity of members of a set. The symbols we commonly use are derived from those of ancient India and Arabia. Those ancients invented the nine basic symbols of 1 representing an individual finger, 2, 3, 4, 5, 6, 7, 8, 9 representing subsequent sets of fingers (digits) and symbol 10 for the full set of fingers.

The *value* of a number is the size of the collection of individuals that it represents. An individual has a value of one unit.

To *count* is to use a systematic process to find the *value* of a collection. Methods have included such as using the fingers, tally marks, counters, number symbols or electronic calculators.

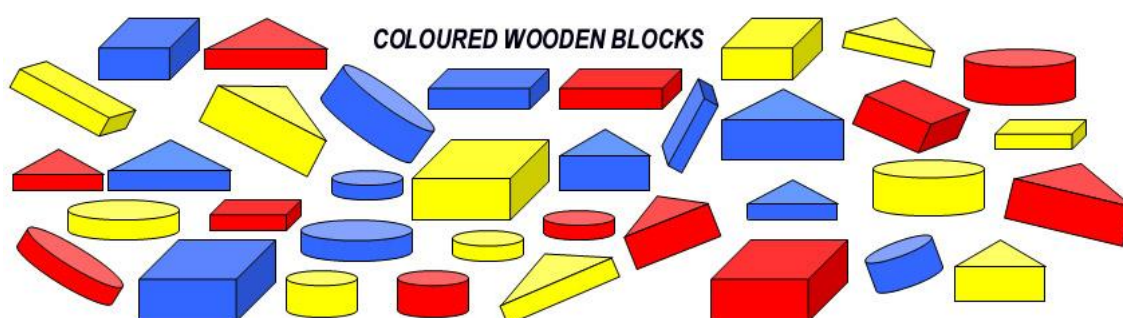
To *process* is to apply a logical rule to change an individual or to change the members of a set. In logical processes an action (a cause) has a consistent result (an effect). This predictability enables mathematics to be used to provide workplace solutions other than by trial and error.

Basic *calculation* is to use counters or numbers in the processes of *addition* (add), *subtraction* (take away), *multiplication* (times) and *division* (share).

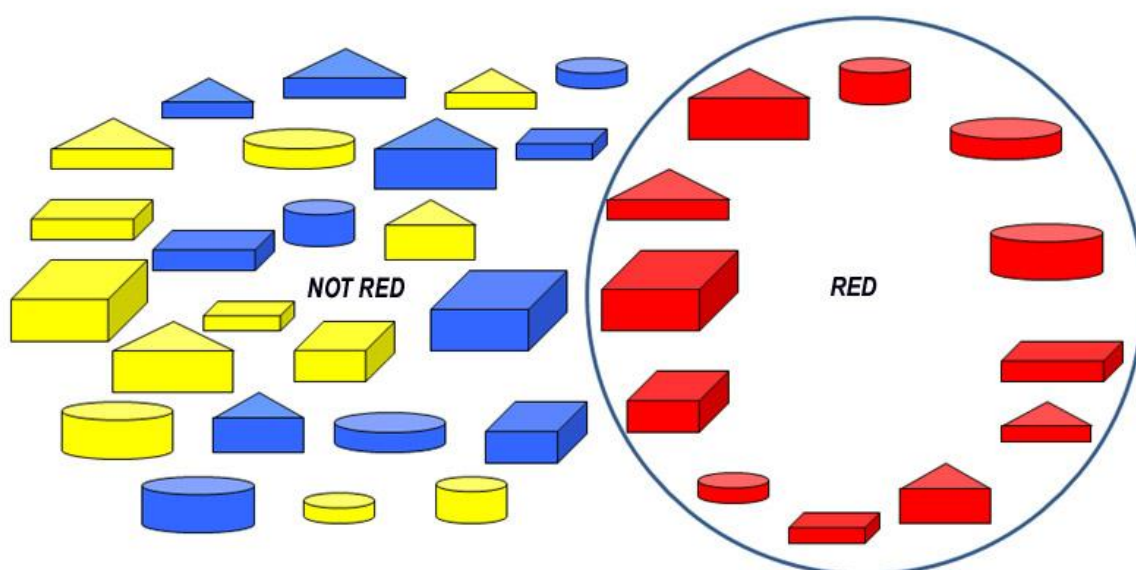
## Sets and sorting

To understand our world we seek consistent rules where actions have predictable results – we look for a cause leading to a repeatable affect – we apply logic.

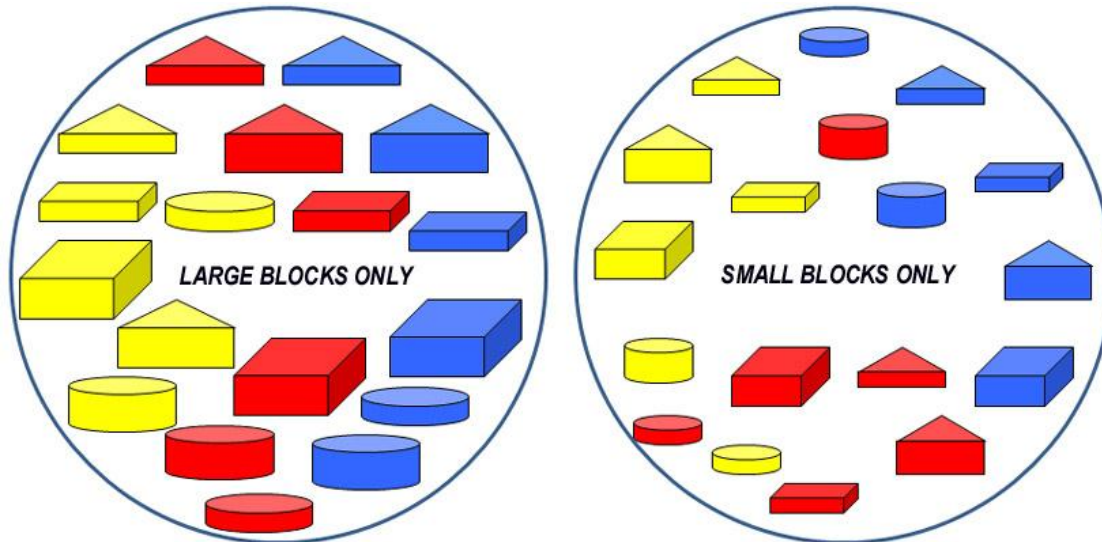
Our world is filled with many things - some have similar and some dissimilar features. An *attribute* is a defining feature of a thing. The picture below shows a collection of things that have the common *attribute* of being coloured wooden blocks but differing *attributes* of colour, shape, thickness or size. *Attributes* define what makes a thing an individual. For instance, there is only one circle that is red, thick and large.



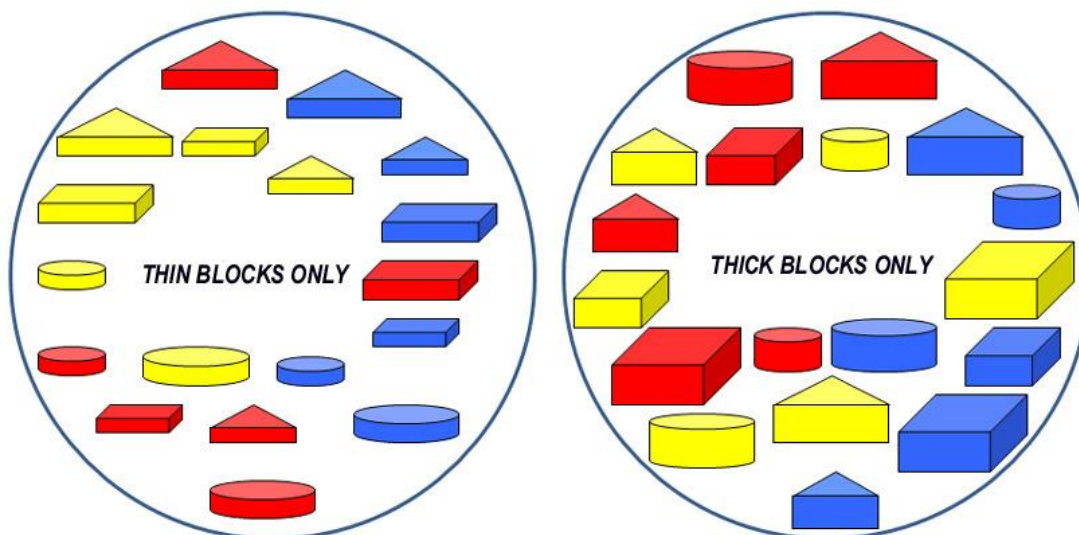
*Attributes* allow us to compare one thing with another, make rules to classify them into logical collections and *sort* them into *sets* (groups of individuals with similar attributes). The chaotic pile of blocks above can be logically sorted and made sense of in a number of ways. All the red blocks have been selected to form a red set, leaving a remainder of those that do not have the red attribute (are yellow or blue).



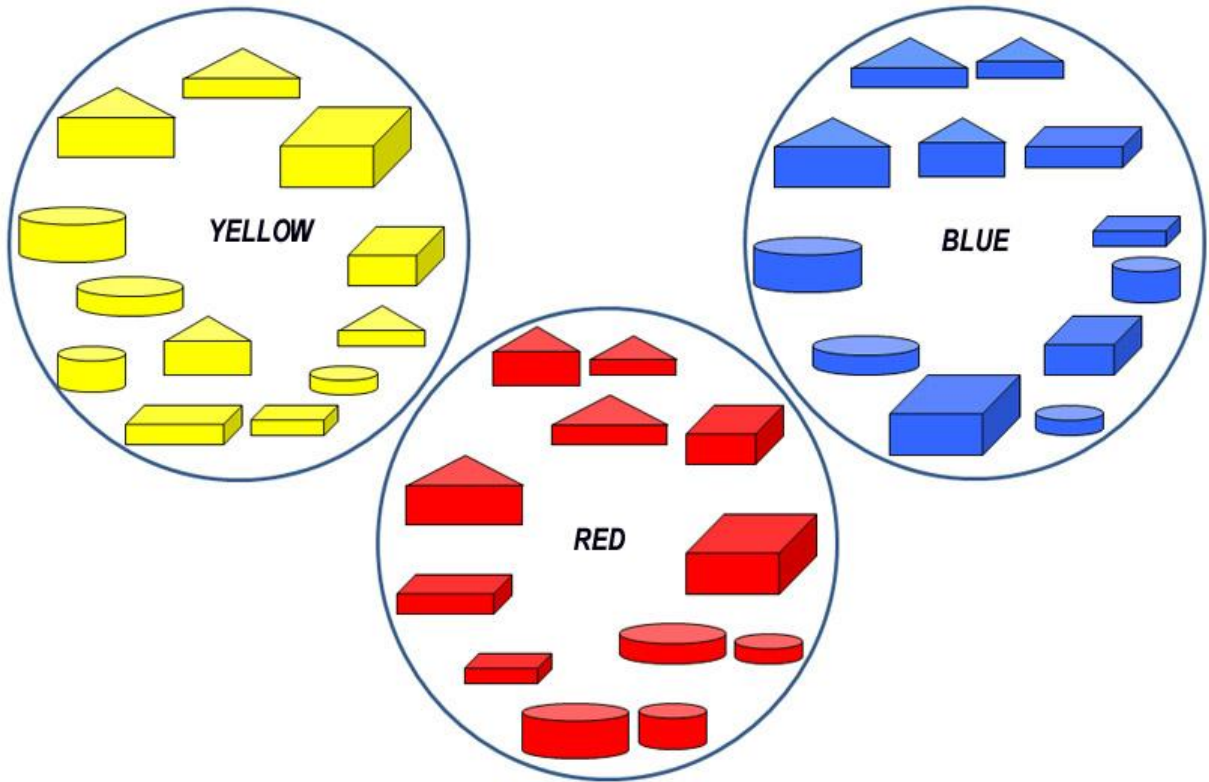
However we could just as easily have created different sets by using different attributes to separate the wooden blocks into like groups. The logical rule we have used below is to select for each set on the basis of size only – large or small attribute.



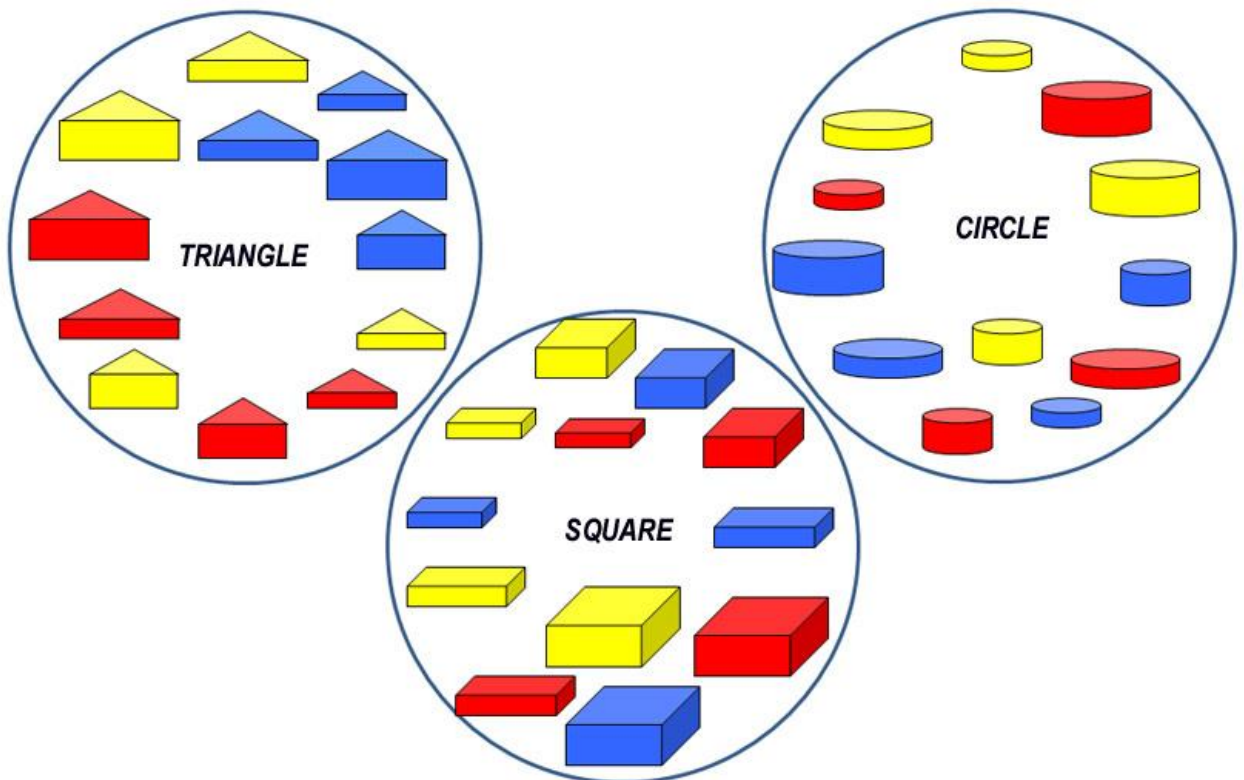
Or we could have created other sets by using other attributes of the wooden blocks. The logical rule we have used below is to select on the basis of thickness only – thin or thick attribute.







Other sets are created by using the other attributes of colour or shape.

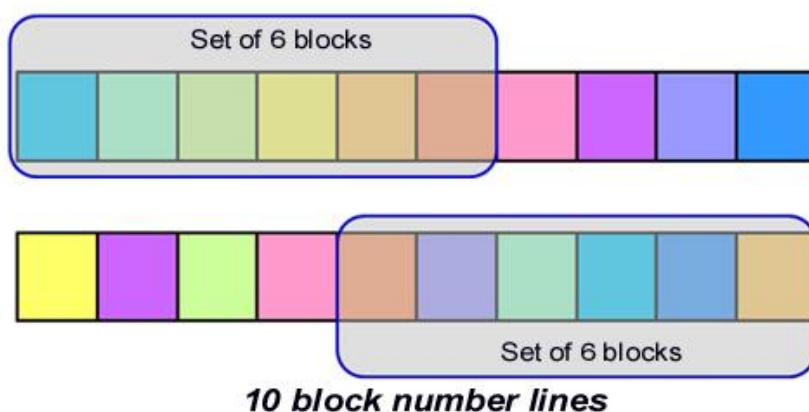


In summary, *attributes* define what makes an individual. For instance, there is only one circle that is red, thick and large. *Attributes* also define the members that can make up a logical group so enabling set size to be compared. For instance, the size of each shape set above compares to a set of 10 of our fingers plus 2 of our toes.

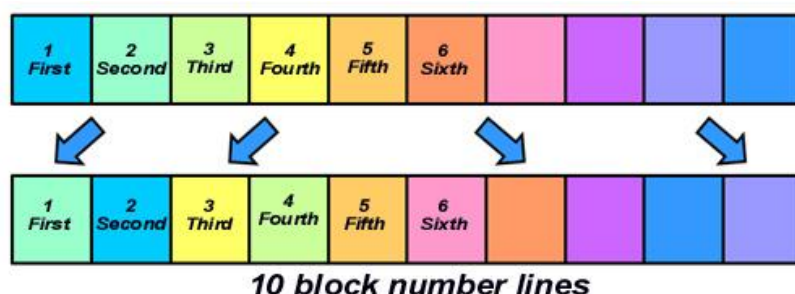
## 1.2 Number, value, and counting

A *number* is a concept that represents the *value* of a *set* (collection). The number symbols used today are derived from those developed in ancient India and Arabia. 1 represents a finger or individual (a digit), 2, 3, 4, 5, 6, 7, 8, 9 represent the *quantity* or *sequence* of digits and 10 represents a full set of digits counted on both hands.

The *value* of a number describes the *quantity* of individuals in collection or the *sequence* of an individual's position within a set - whether it is first, second, third etc.



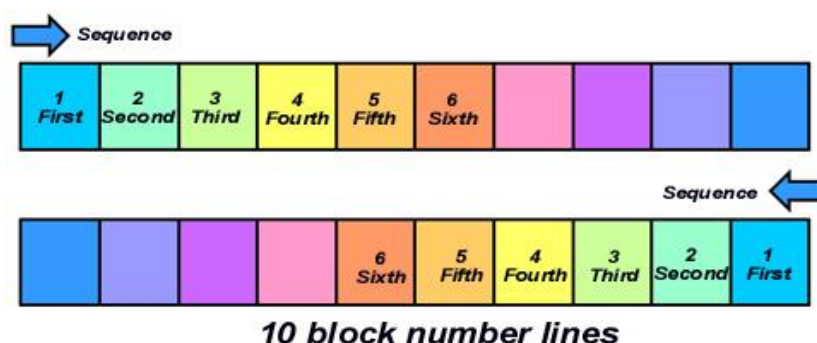
Shown above are *quantities* of six block sets that have been separated from the *number lines* (each of ten blocks). Note that though both top and bottom sets contain 6 blocks, the blocks themselves are different. Number 6 is not the exclusive name of either set, but a description of a set's attributes - they each contain six blocks.



Shown above are two number line *sequences* where the blocks have been rearranged. The name of top row yellow block number 4 is not a constant. If it is re-positioned in the bottom row then it is renamed as block number 3.



Shown below are two number line *sequences* where the blocks have been rearranged. This time the lower line has been reversed. Unlike the previous number lines, we can give each block a constancy of name if we apply a constancy rule - to reverse the starting point of our sequence from left to right, to right to left.



It is natural to use numbers as names from an object's place in a group, such as 1<sup>st</sup> finger, 2<sup>nd</sup> finger and 3<sup>rd</sup> finger. However, name and number are different ideas. *Constancy of number* relates to the logical rule not the name temporarily given to an object. An ability to trust in learnt mathematical rules and before relating back to real objects is needed in order to efficiently count, process and eventually to calculate.

To *count* is to use a systematic process to find the *value* of a collection. Methods include using the fingers, measuring, striking tally marks, use of counters, number lines, mental arithmetic, tables and electronic calculators.

### Tally - an historical perspective

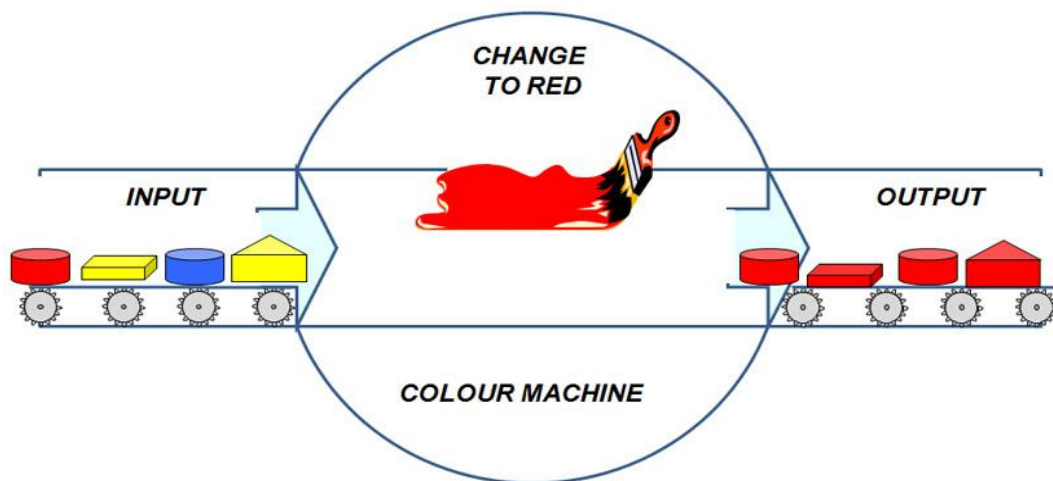
With the fingers on one hand we can communicate the idea of up to five things. Counting is seen in ancient tally marks scratched onto clay tablets to represent flocks of sheep or chalked as tally marks of present day stevedores. As each collection is gathered (recorded as groups of scratches) the marks are slashed through to reckon a total - below records two sets of 5 sheep and another 4.



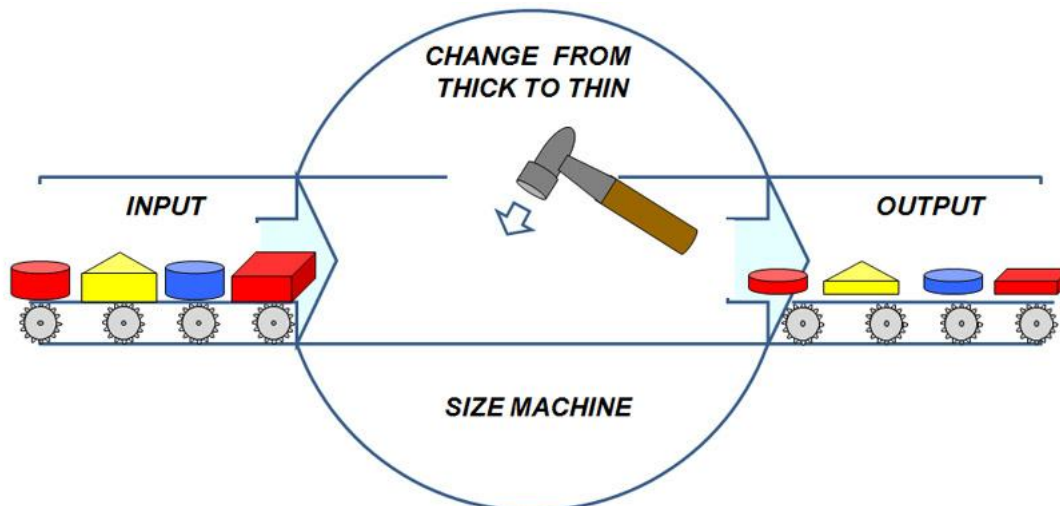
*Tallying* requires only observation skills to accurately strike the marks, unlike processing (to apply a logical rule predicting a result). Processing is explained below.

### Processing

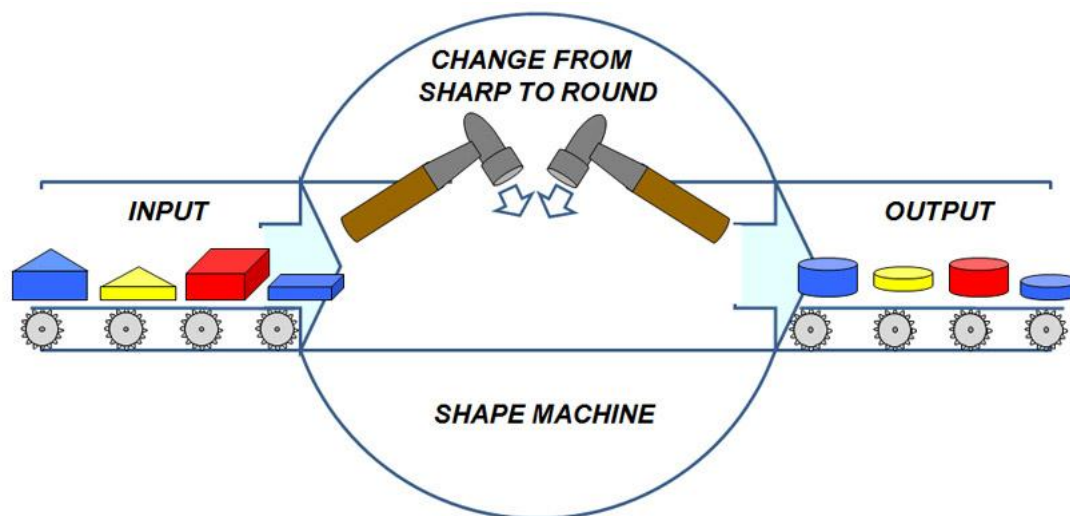
Once you have defined the *attribute/s* of an individual, a rule can be invented to *process* or logically change that *attribute/s*. The processing rule used by the *colour machine* shown below is to paint (change to red) anything passed through the *input* to the *output*. All other attributes remain unchanged (the shape, size and thickness).



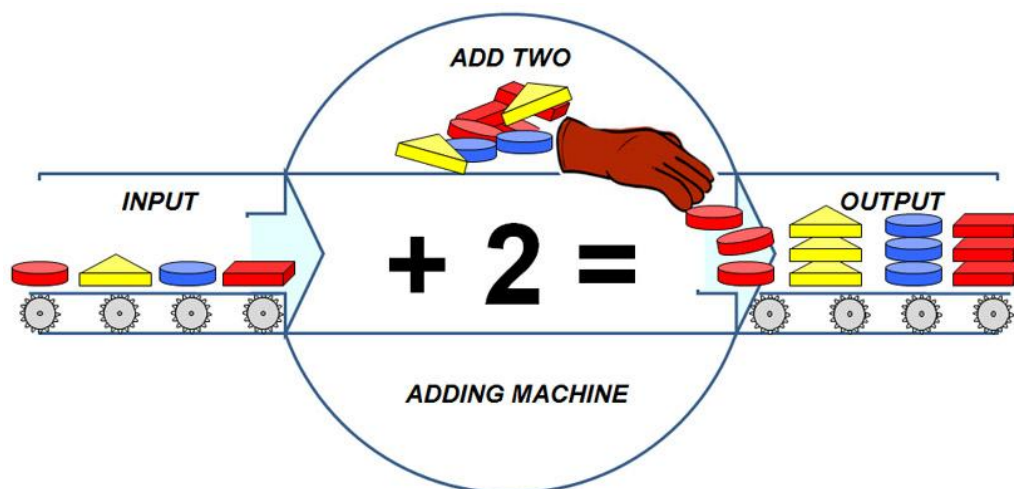
However we could just as easily have created different *output* by changing the processing rule. The rule used by the logical *thickness machine* shown below is to hammer (change to thin) anything passed through the *input* to the *output*. All other attributes remain unchanged (the shape, size and colour).



Or we could just as easily have created different *output* by another processing rule. The rule used by the logical *rounding machine* shown below is to hammer (change to round) anything passed through the *input* to the *output*. All other attributes remain unchanged (the thickness, size and colour).



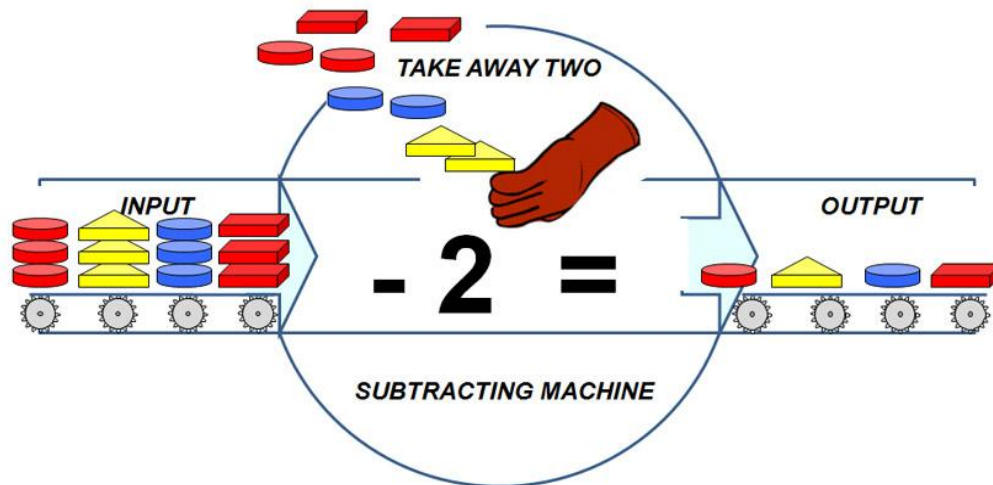
Similarly the rule used by the logical *copy machine* shown below is to make two additional copies of anything passed through the *input* to the *output*. All attributes remain unchanged as the additional units are exact copies (clones).



We can represent this adding machine as a *sum* using the processing symbols of **+** for plus (adding) and **=** for equals (the result). For instance, for each block inputted:

$$1 + 2 = 3$$

1 (red square)	+ 2 (red squares)	= 3 (red squares)
1 (blue circle)	+ 2 (blue circles)	= 3 (blue circles)
1 (yellow triangle)	+ 2 (yellow triangles)	= 3 (yellow triangles)
1 (red circle)	+ 2 (red circles)	= 3 (red circles)



Likewise the *removing machine* above that takes away two of anything passed through it can be represented by a *subtraction* using the symbols - for subtract (take away) and = for equals (the result). For instance, for each group of blocks inputted:

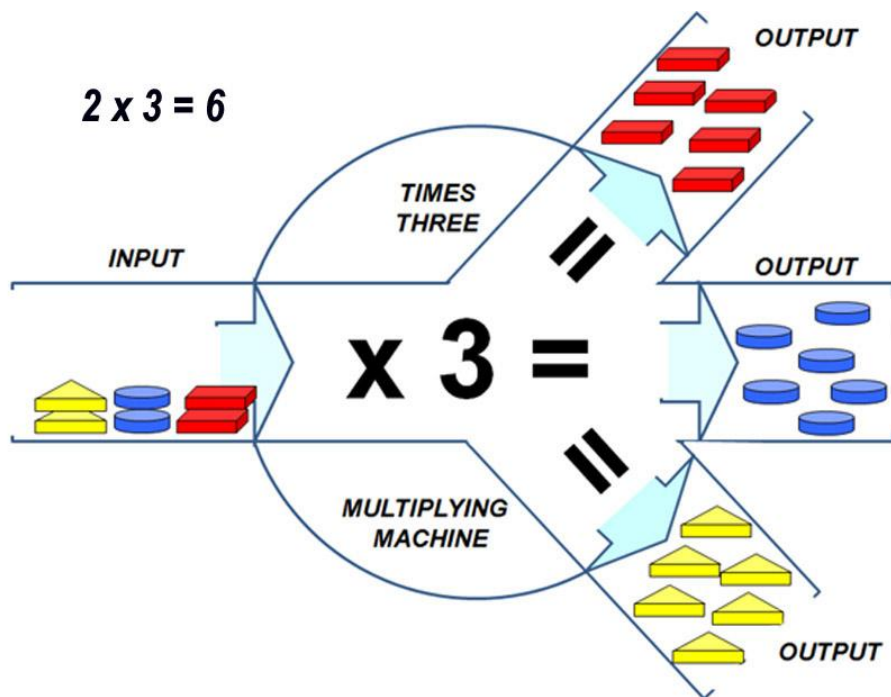
$$3 - 2 = 1$$

3 (red squares)	- 2 (red squares)	= 1 (red square)
3 (blue circles)	- 2 (blue circles)	= 1 (blue circle)
3 (yellow triangles)	- 2 (yellow triangles)	= 1 (yellow triangle)
3 (red circles)	- 2 (red circles)	= 1 (red circle)

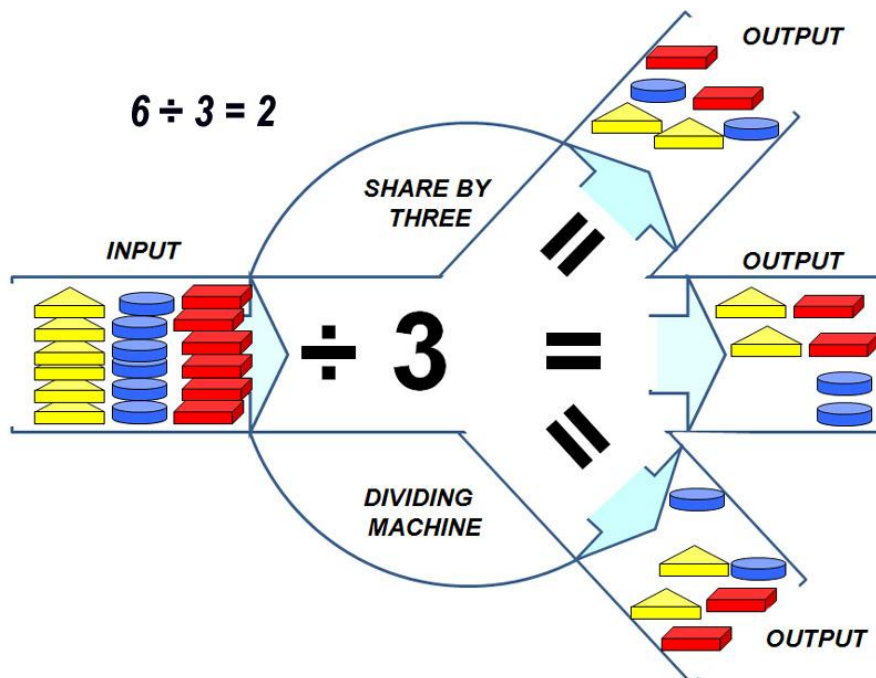
The logical *multiplying machine* shown below that trebles anything passed through it can be represented by a *multiplication* using the symbols x for multiply (times) and = for equals (the answer). For instance, for each block inputted:

$$1 \times 3 = 3$$

2 (yellow triangles)	x	3	= 6 (yellow triangle)
2 (blue circles)	x	3	= 6 (blue circle)
2 (red squares)	x	3	= 6 (red square)



The logical *sharing machine* shown below can be represented by a *division* using the symbols  $\div$  for divide (share) and  $=$  for equals (the answer). For instance, for each group of blocks inputted:




## Tables and examples

Add +	Take away -		Times x	Share ÷
1 + 1 = 2	1 - 1 = 0	5 - 5 = 0	1 x 1 = 1	1 ÷ 1 = 1
1 + 2 = 3	2 - 1 = 1	6 - 5 = 1		
1 + 3 = 4	3 - 1 = 2	7 - 5 = 2	2 x 2 = 4	2 ÷ 2 = 1
1 + 4 = 5	4 - 1 = 3	8 - 5 = 3	2 x 3 = 6	4 ÷ 2 = 2
1 + 5 = 6	5 - 1 = 4	9 - 5 = 4	2 x 4 = 8	6 ÷ 2 = 3
1 + 6 = 7	6 - 1 = 5		2 x 5 = 10	8 ÷ 2 = 4
1 + 7 = 8	7 - 1 = 6	6 - 6 = 0		10 ÷ 2 = 5
1 + 8 = 9	8 - 1 = 7	7 - 6 = 1	3 x 2 = 6	
1 + 9 = 10	9 - 1 = 8	8 - 6 = 2	3 x 3 = 9	3 ÷ 3 = 1
		9 - 6 = 3		6 ÷ 3 = 2
2 + 2 = 4	2 - 2 = 0	7 - 7 = 0		9 ÷ 3 = 3
2 + 3 = 5	3 - 2 = 1	8 - 7 = 1	4 x 2 = 8	
2 + 4 = 6	4 - 2 = 2	9 - 7 = 2		
2 + 5 = 7	5 - 2 = 3		5 x 2 = 10	4 ÷ 4 = 1
2 + 6 = 8	6 - 2 = 4	8 - 8 = 0		8 ÷ 4 = 2
2 + 7 = 9	7 - 2 = 5	9 - 8 = 1		
2 + 8 = 10	8 - 2 = 6	9 - 9 = 0		5 ÷ 5 = 1
	9 - 2 = 7			10 ÷ 5 = 2
3 + 3 = 6	3 - 3 = 0	10 - 1 = 9		
3 + 4 = 7	4 - 3 = 1	10 - 2 = 8		
3 + 5 = 8	5 - 3 = 2	10 - 3 = 7		
3 + 6 = 9	6 - 3 = 3	10 - 4 = 6		
3 + 7 = 10	7 - 3 = 4	10 - 5 = 5		
	8 - 3 = 5	10 - 6 = 4		
	9 - 3 = 6	10 - 7 = 3		
		10 - 9 = 2		
4 + 4 = 8	4 - 4 = 0	10 - 9 = 1		
4 + 5 = 9	5 - 4 = 1			
4 + 6 = 10	6 - 4 = 2			
	7 - 4 = 3			
5 + 5 = 10	8 - 4 = 4			
	9 - 4 = 5			
<b>Plus</b>	<b>Minus</b>		<b>Multiply</b>	<b>Divide</b>



## 1.3 Numbering systems and number base

All numbering systems employ a base set of symbols. The commonly used, base ten ( $1_{10}$ ), imitates a full set of fingers (our digits). Combinations of the nine symbols of 1, 2, 3, 4, 5, 6, 7, 8, 9 (representing each value) and 0 (representing zero or tens of units) can represent any number. Two sets are then 20, three 30, and so on.



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Others bases are used when convenient, such as base two or binary system used in computer code. Below the tally marks show *eleven* marked in sets of two (*Base*  $_2$ ) and *twenty three* marked off in both sets of five (*Base*  $_5$ ) and in sets of ten (*Base*  $_{10}$ ).

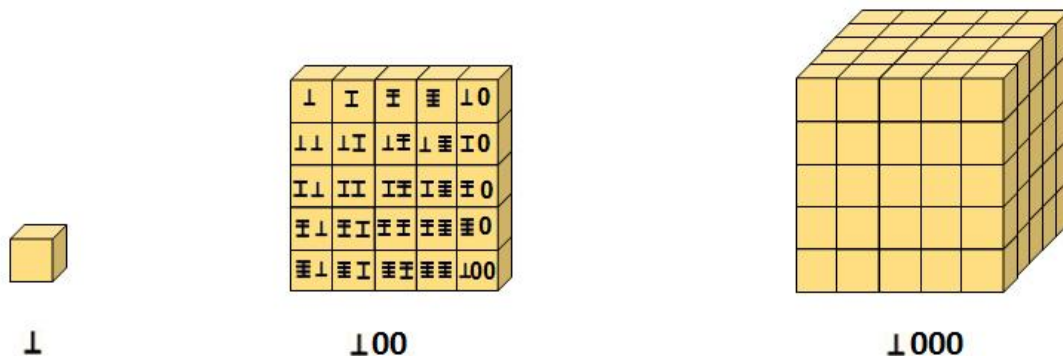
 = *Eleven*  
 = *Twenty three*  
 = *Twenty three*

### Base Five ( $1_5$ )

Base five numbering system imitates a set of fingers on one hand only and (example below) uses combinations of four symbols and a zero to express all numbers.



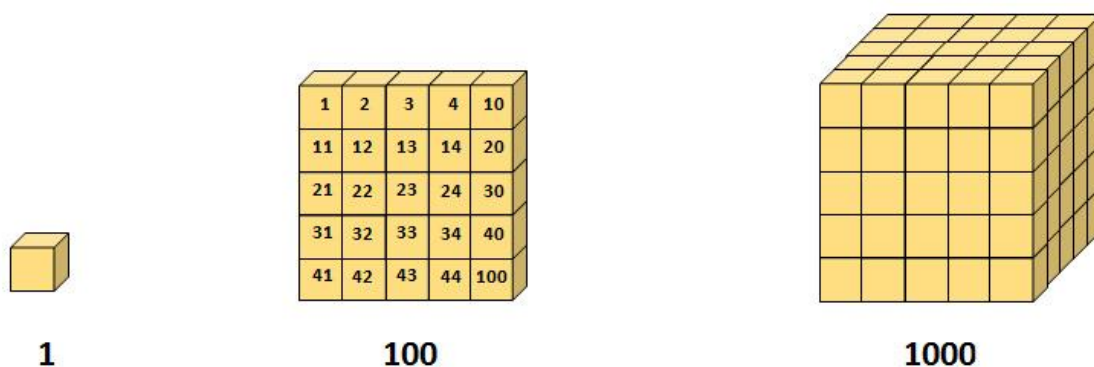
In base<sub>5</sub>, we have an individual digit, or we can build a square or a cube of with groups of individuals. The total value of each pile is logically numbered as shown below using the invented symbols of slashes on sticks.



In base<sub>5</sub> using our invented symbols we can express sets, values and a number to represent each value as:




<b>Base Sets</b>	<b>Digits Value</b>	<b>Number Symbol</b>
Two sets are written (digits of both hands)		<b>10</b>
Three sets are written		<b>10</b>
Five sets are written		<b>100</b>

However, we could have used the Indus-Arabic symbols of 1, 2, 3, 4, and 0 to express our numbers. To represent the thumb a zero is now partnered with symbol 1 so the number symbol of 10 now represents a full set of five digits.



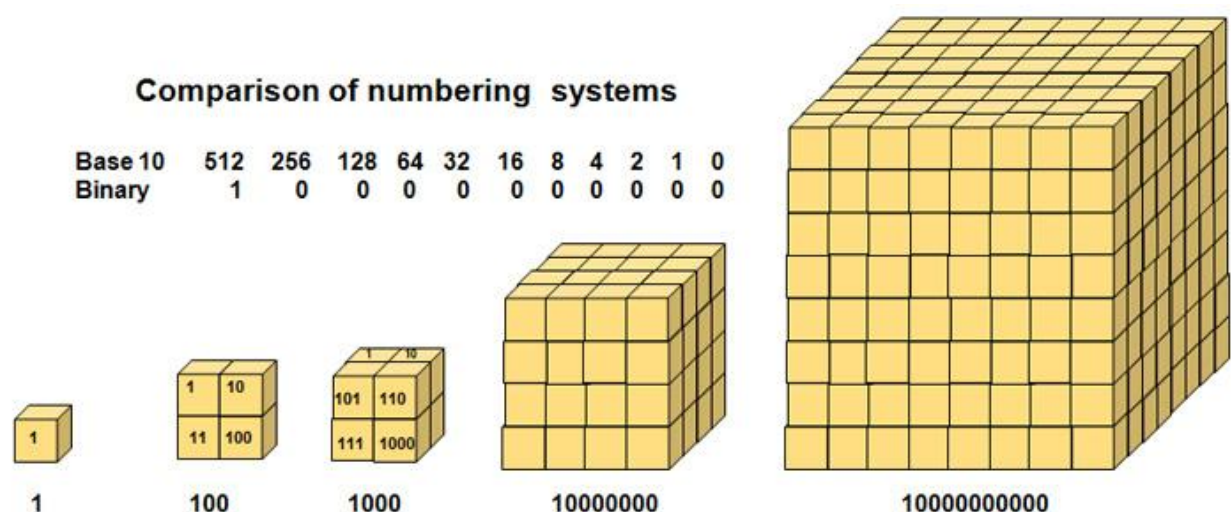
Take care that you only use one base system at a time, as each base attributes a different value to its number symbols. Base<sub>5</sub> number 1000<sub>5</sub> does not have the same value as Base<sub>10</sub> number 1000<sub>10</sub>. We use base<sub>10</sub> so universally that we rarely annotate with the base system symbol or think about the bases of our numbering systems.

In base five using conventional Arabic symbols we can express sets, values and a number to represent each value as:

<b>Base Sets</b>	<b>Digits Value</b>	<b>Base 5</b>	<b>Base 10</b>
Two sets are written (digits of both hands)		<b>20</b>	<b>10</b>
Three sets are written		<b>30</b>	<b>15</b>
Five sets are written		<b>100</b>	<b>25</b>

## Base Two (1<sub>2</sub>)

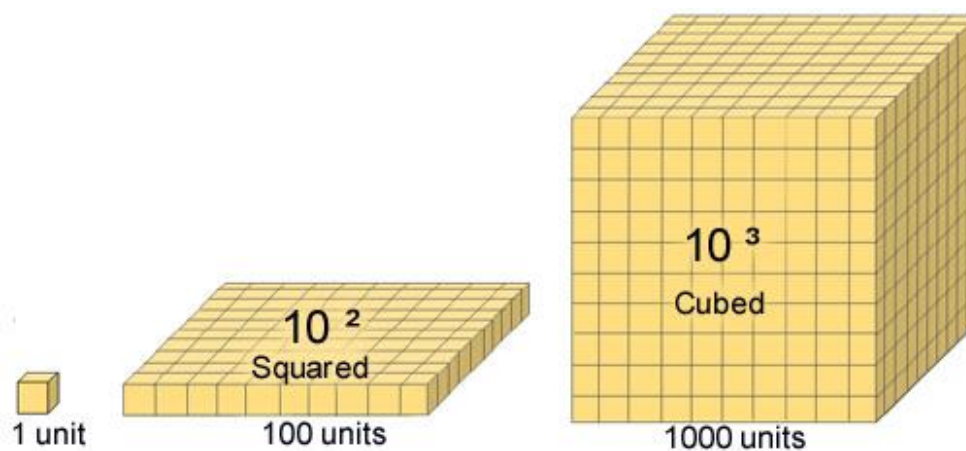
The Base-two number system (binary numeral system) is used for computer code because electronic switches can easily read and store two states, on or off (twin voltage may be used by some systems). To these states are given the symbols 1 and 0.



A conversion from a binary number's value to its equivalent base<sub>10</sub> value can be made with the table shown above. Binary values are derived from sets of two and progressive multiples. These values of 64, 512, 1028, and 2056 megabytes are used to specify the Random Access Memory power of a computer's micro-processor.

## 1.4 Scientific notation - integral exponents and roots

Understand Chapter Two before reading this section. To reduce the zeros required to write the very large and very small numbers that engineers use, a short hand termed of *numbers to a power* (integral exponents) is used. These symbols are written by the right hand top of a number. Shown in the examples below, *ten* is the number and the power is to *two* and to *three*, commonly called squared or cubed.



*Described as ten squared and written as  $10^2$  is calculated  $10 \times 10 = 100$*

*Described as ten cubed and written as  $10^3$  is calculated  $10 \times 10 \times 10 = 1000$*

Similarly,

*Ten to the fourth power is  $10^4$  is calculated as  $10 \times 10 \times 10 \times 10 = 10,000$*

*Ten to the fifth power is  $10^5$  is calculated as  $10 \times 10 \times 10 \times 10 \times 10 = 100,000$*

Any number can be written to a power as in the examples below:

*Three squared is written as  $3^2$  and calculated as  $3 \times 3 = 9$*

*Four cubed is written as  $4^3$  and calculated as  $4 \times 4 \times 4 = 32$*

*Five to the fourth power is  $5^4$  and calculated as  $5 \times 5 \times 5 \times 5 = 625$*

*Six to the fifth power is  $6^5$  and calculated as  $6 \times 6 \times 6 \times 6 \times 6 = 1296$*

When calculating with multiples of 10, the *power* defines the number of zeros (0):

*Ten to the power of two has two zeros*                      *(100 = a hundred),*  
*Ten to the power of three has three zeros*                      *(1000 = a thousand)*  
*Ten to the power of six has six zeros*                      *(1,000,000 = a million)*  
*Ten to the power of twelve has twelve zeros*                      *(1,000,000,000,000 = a billion)*

\* Note France & US use one thousand times one million to define a US billion)

For instance, below both numbers can be written to a power.

$$\begin{aligned} 257 &= 2.57 \times 100 = 2.57 \times 10^2 \\ 2,570,000 &= 2.57 \times 1,000,000 = 2.57 \times 10^6 \end{aligned}$$

The convenience of exponents is further seen in expressing the concept of infinity (going on forever without end). The simple expression of ten to the power of infinity as below would otherwise require an unending task of writing 0 after 0 without end.

To power of infinity  $10^\infty$  is defined as  $10 \times 10$  continually repeated

Exponents also simplify otherwise tedious calculations and lessen the chance of error in the positioning of a decimal point. As below, exponents can be written as powers of ten or as decimals by use of the minus sign.

$$\begin{array}{ll} 10^5 = 100,000 & 10^{-5} = 0.00001 \\ 10^4 = 10,000 & 10^{-4} = 0.0001 \\ 10^3 = 1000 & 10^0 = 1 & 10^{-3} = 0.001 \\ 10^2 = 100 & & 10^{-2} = 0.01 \\ 10^1 = 10 & & 10^{-1} = 0.1 \end{array}$$

For consistency using decimal place, the following rule is used:

Place the decimal point immediately to the right of the first non zero digit.  
 Compare the number of digits between the new decimal point and the original.  
 This number is the value of the exponent.

$$\begin{aligned} 0.00257 &= 2.57 \times 0.001 = 2.57 \times 10^{-3} \\ 0.257 &= 2.57 \times 0.1 = 2.57 \times 10^{-1} \end{aligned}$$



## Roots (inverse powers)

The root of a number is that value that when multiplied by itself for specified repeat instances will be equal to the number. It is the inverse (reversed process) of a power, for instance:

The *square root* of 100 written as  $\sqrt{100}$  is 10 because  $10 \times 10 = 100$   
The *cubed root* of 1000 written as  $\sqrt[3]{1000}$  is 10 because  $10 \times 10 \times 10 = 1000$

All numbers have a root as in the examples below:

The *cubed root* of 8 written as  $\sqrt[3]{8}$  is 2 because  $2 \times 2 \times 2 = 8$   
The *fouth root* of 81 written as  $\sqrt[4]{81}$  is 3 because  $3 \times 3 \times 3 \times 3 = 81$   
The *fifth root* of 7776 written as  $\sqrt[5]{7776}$  is 6 because  $6 \times 6 \times 6 \times 6 \times 6 = 7776$

Some numbers roots and number are the same, for example numbers 0 and 1 whose roots are 0 and 1:

The *square root* of 0 written as  $\sqrt{0}$  is 0 because  $0 \times 0 \times 0 = 0$   
The *cubed root* of 1 written as  $\sqrt[3]{1}$  is 1 because  $1 \times 1 \times 1 = 1$

The roots of some numbers are called *irrational numbers* because they never resolve as a whole number, for example the roots of 2, 5 and 7:

The *square root* of 2 written as  $\sqrt{2}$  is **1.414213562**  
The *cubed root* of 5 written as  $\sqrt[3]{5}$  is **2.236067978**  
The *fouth root* of 81 written as  $\sqrt[4]{7}$  is **2.645751311**

## Fractional exponents

Some engineering problems require solving problems that involve the use of fractional powers. While these are easily solved following the stepped instructions with a scientific calculator, the following examples are shown in explanation.



Exponents can be written as fractions. In these cases exponent  $\frac{1}{2}$  indicates the power of 1 and square root of the number. Similarly exponent  $\frac{1}{3}$  indicates the cube root,  $\frac{1}{4}$  the fourth root,  $\frac{1}{8}$  the eighth root, and so on.

$$\begin{aligned}
 16^{\frac{1}{2}} &= 16^{0.5} = 4 & (16 &= 4 \times 4) \\
 216^{\frac{1}{3}} &= 216^{0.333} = 6 & (216 &= 6 \times 6 \times 6) \\
 81^{\frac{1}{4}} &= 81^{0.25} = 3 & (81 &= 3 \times 3 \times 3 \times 3) \\
 256^{\frac{1}{8}} &= 256^{0.125} = 2 & (256 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)
 \end{aligned}$$

In mathematical terms, the fractional exponents indicate that the numerator will raise a number to that power and the denominator takes it to that root.

$$\begin{aligned}
 10^{\frac{2}{3}} &= 10^{0.666} = \sqrt[3]{10^2} = 4.6 & \text{as } (10^2 = 100) & \text{and } (\sqrt[3]{100} = 4.64 \times 4.64 \times 4.64) \\
 8^{\frac{2}{3}} &= 8^{0.666} = \sqrt[3]{8^2} = 4 & \text{as } (8^2 = 64) & \text{and } (\sqrt[3]{64} = 4 \times 4 \times 4) \\
 10^{\frac{3}{4}} &= 10^{0.750} = \sqrt[4]{10^3} = 5.62 & \text{as } (10^3 = 1000) & \text{and } (\sqrt[4]{1000} = 5.6 \times 5.6 \times 5.6 \times 5.6) \\
 6^{\frac{3}{5}} &= 6^{0.600} = \sqrt[5]{6^3} = 2.93 & \text{as } (6^3 = 216) & \text{and } (\sqrt[5]{216} = 2.9 \times 2.9 \times 2.9 \times 2.9 \times 2.9) \\
 10^{\frac{3}{5}} &= 10^{0.600} = \sqrt[5]{10^3} = 4 & \text{as } (10^3 = 1000) & \text{and } (\sqrt[5]{1000} = 4 \times 4 \times 4 \times 4 \times 4) \\
 7^{\frac{3}{4}} &= 7^{0.750} = \sqrt[4]{7^3} = 4.3 & \text{as } (7^3 = 343) & \text{and } (\sqrt[4]{343} = 4.3 \times 4.3 \times 4.3 \times 4.3)
 \end{aligned}$$

## Chapter 2: Calculations

Using the fingers to count or share more than ten units is difficult. This section describes other ways to solve *arithmetic problems* when using base ten, the commonly used numbering system. (See more about other bases in [Section 1.3](#)).

The four principle operations are denoted by symbols (signs) and are named:

+	Plus	to add	or to sum	by addition
-	Minus	to subtract	or to take away	by a subtraction
x	Times	to multiply	or times	by a multiplication
÷	Divide by	to divide	or to share	by a division

A calculation is a *arithmetical problem* with its *solution* (its answer). Calculations can be *expressed* (written down in maths shorthand) using an *equation*. The problem is written on the left, the solution on the right with an *equals* symbol (=) between them. In a balanced equation, each side of the equals sign must have the same total. (See more on equations and algebra in [Section 2.4](#)). Simple equations are shown below:

$$1 + 1 = 2 \quad \text{or} \quad 3 - 1 = 2 \quad \text{or} \quad 2 \times 2 = 4 \quad \text{or} \quad 4 \div 2 = 2$$

*1 plus 1 equals 2      3 minus 1 equals 2      2 times 2 equals 4      4 divide by 2 equals 2*

### 2.1 Addition, subtraction, multiplication and division

Various methods used to find solutions are described below.

#### Using the Number Line method and mental arithmetic

A scale or ruler is a *number line* when used as a visual aid in addition (counting out), subtraction (finding a remainder), division (sharing) and multiplication (times).

In your mind cut the stick below into 12 slices and then count the slices out to share with ten persons. Ten can be evenly shared but two slices are left over. We have used the *number line* below to record our sharing and the remainder that was left.



What we have done can be expressed by the equations:

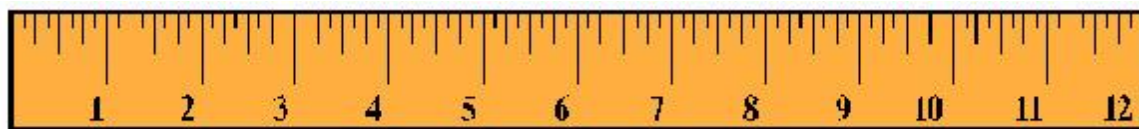
<b>Addition</b>	<b>Subtraction</b>
$1+1+1+1+1+1+1+1+1+1=10$ <i>shared orange</i>	$12 - 10 = 2$ <i>remainder brown</i>

Multiplication and division could also be visualised using the number line. The 12 slices could be counted out in sets (See 1.1 Sets and sorting):

<b>2 sets of</b>	<b>six slices</b> can be written as	$2 \times 6 = 12$	or	$12 \div 6 = 2$
<b>3 sets of</b>	<b>four slices</b> can be written as	$3 \times 4 = 12$	or	$12 \div 3 = 4$
<b>4 sets of</b>	<b>three slices</b> can be written as	$4 \times 3 = 12$	or	$12 \div 4 = 3$
<b>6 sets of</b>	<b>two slices</b> can be written as	$6 \times 2 = 12$	or	$12 \div 2 = 6$

While this scale is marked in twelve units, each unit could be further divided into 8 sub units to solve more complex problems (by counting up the units), as below:

1 units	x	8 sub-units	=	8 sub-units
4 units	x	8 sub-units	=	32 sub-units
12 units	x	8 sub-units	=	96 sub-units



The number line works well for small numbers but as it relies on counting out every unit it is slow for complex problems.

### Using the Times Tables method

Fast solution to maths problems can be looked up or learnt as the *times tables* below. (also see *addition tables* [Section 1.2](#)). They can also be used to find division solutions. It can be seen that one side of each equation is equal to the other. For example, in the *times x 4 table* equation of below  $8 \times 4 = 32$ , it is also true that 32 shared among 4 will have a solution of 8.

$$8 \times 4 = 32 \quad \text{or} \quad 32 \div 4 = 8$$

Times x 1	Times x 2	Times x 3	Times x 4	Times x 5	Times x 6
1 x 1 = 1	1 x 2 = 2	1 x 3 = 3	1 x 4 = 4	1 x 5 = 5	1 x 6 = 6
2 x 1 = 2	2 x 2 = 4	2 x 3 = 6	2 x 4 = 8	2 x 5 = 10	2 x 6 = 12
3 x 1 = 3	3 x 2 = 6	3 x 3 = 9	3 x 4 = 12	3 x 5 = 15	3 x 6 = 18
4 x 1 = 4	4 x 2 = 8	4 x 3 = 12	4 x 4 = 16	4 x 5 = 20	4 x 6 = 24
5 x 1 = 5	5 x 2 = 10	5 x 3 = 15	5 x 4 = 20	5 x 5 = 25	5 x 6 = 30
6 x 1 = 6	6 x 2 = 12	6 x 3 = 18	6 x 4 = 24	6 x 5 = 30	6 x 6 = 36
7 x 1 = 7	7 x 2 = 14	7 x 3 = 21	7 x 4 = 28	7 x 5 = 35	7 x 6 = 42
8 x 1 = 8	8 x 2 = 16	8 x 3 = 24	8 x 4 = 32	8 x 5 = 40	8 x 6 = 48
9 x 1 = 9	9 x 2 = 18	9 x 3 = 27	9 x 4 = 36	9 x 5 = 45	9 x 6 = 54
10 x 1 = 10	10 x 2 = 20	10 x 3 = 30	10 x 4 = 40	10 x 5 = 50	10 x 6 = 60
Times x 7	Times x 8	Times x 9	Times x 10	Times x 11	Times x 12
1 x 7 = 7	1 x 8 = 8	1 x 9 = 9	1 x 10 = 10	1 x 11 = 11	1 x 12 = 12
2 x 7 = 14	2 x 8 = 16	2 x 9 = 18	2 x 10 = 20	2 x 11 = 22	2 x 12 = 24
3 x 7 = 21	3 x 8 = 24	3 x 9 = 27	3 x 10 = 30	3 x 11 = 33	3 x 12 = 36
4 x 7 = 28	4 x 8 = 32	4 x 9 = 36	4 x 10 = 40	4 x 11 = 44	4 x 12 = 48
5 x 7 = 35	5 x 8 = 40	5 x 9 = 45	5 x 10 = 50	5 x 11 = 55	5 x 12 = 60
6 x 7 = 42	6 x 8 = 48	6 x 9 = 54	6 x 10 = 60	6 x 11 = 66	6 x 12 = 72
7 x 7 = 49	7 x 8 = 56	7 x 9 = 63	7 x 10 = 70	7 x 11 = 77	7 x 12 = 84
8 x 7 = 56	8 x 8 = 64	8 x 9 = 72	8 x 10 = 80	8 x 11 = 88	8 x 12 = 96
9 x 7 = 63	9 x 8 = 72	9 x 9 = 81	9 x 10 = 90	9 x 11 = 99	9 x 12 = 108
10 x 7 = 70	10 x 8 = 80	10 x 9 = 90	10 x 10 = 100	10 x 11 = 110	10 x 12 = 120

## Using Short calculations - simple addition, subtraction, multiplication, division

**Short addition** calculations are written as columns to sum in an answer box below:

$$\begin{array}{r} \text{UNIT} \\ 5 \\ +3 \\ \hline 8 \end{array}$$

$$\begin{array}{r} \text{UNIT} \\ 4 \\ +5 \\ \hline 9 \end{array}$$

$$\begin{array}{r} \text{UNIT} \\ 7 \\ +2 \\ \hline 9 \end{array}$$

$$\begin{array}{r} \text{UNIT} \\ 6 \\ +3 \\ \hline 9 \end{array}$$

Lists can also be added. Note - for numbers of 10 or more the first step is to add the Units column (right hand in blue below), and the second step the left hand Tens:

$$\begin{array}{r} 3 \\ 2 \\ +3 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 4 \\ 1 \\ +2 \\ \hline 7 \end{array}$$

$$\begin{array}{r} \text{TENS UNITS} \\ 4 \\ 1 \text{ } 2 \\ +1 \text{ } 2 \\ \hline 2 \text{ } 8 \end{array}$$

$$\begin{array}{r} \text{TENS UNITS} \\ 1 \text{ } 2 \\ 1 \text{ } 3 \\ +1 \text{ } 0 \\ \hline 3 \text{ } 5 \end{array}$$

**Check your progress addition** -try these sums yourself:

$$\begin{array}{r} 2 \\ 1 \\ +2 \\ \hline ? \end{array}$$

$$\begin{array}{r} 4 \\ 1 \\ +2 \\ \hline ? \end{array}$$

$$\begin{array}{r} 5 \\ 1 \\ +3 \\ \hline ? \end{array}$$

$$\begin{array}{r} 2 \\ 3 \\ +5 \\ \hline ?? \end{array}$$

$$\begin{array}{r} 5 \\ 5 \\ +5 \\ \hline ?? \end{array}$$

$$\begin{array}{r} 6 \\ 10 \\ +12 \\ \hline ?? \end{array}$$

$$\begin{array}{r} 12 \\ 14 \\ +10 \\ \hline ?? \end{array}$$

$$\begin{array}{r} 21 \\ 13 \\ +15 \\ \hline ?? \end{array}$$

**Answers addition** - as below:

$$\begin{array}{r} 2 \\ 1 \\ +2 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 4 \\ 1 \\ +2 \\ \hline 7 \end{array}$$

$$\begin{array}{r} 5 \\ 1 \\ +3 \\ \hline 9 \end{array}$$

$$\begin{array}{r} 2 \\ 3 \\ +5 \\ \hline 10 \end{array}$$

$$\begin{array}{r} 5 \\ 5 \\ +5 \\ \hline 15 \end{array}$$

$$\begin{array}{r} 6 \\ 10 \\ +12 \\ \hline 28 \end{array}$$

$$\begin{array}{r} 12 \\ 14 \\ +10 \\ \hline 36 \end{array}$$

$$\begin{array}{r} 21 \\ 13 \\ +15 \\ \hline 49 \end{array}$$

**Short subtraction** calculations can also be written in the same format as below.

Note - for numbers of 10 or more the first step is to add the Units column (U), and the second step adds the Tens column (T):

$$\begin{array}{r} 5 \\ -3 \\ \hline 2 \end{array}$$

$$\begin{array}{r} 6 \\ -5 \\ \hline 1 \end{array}$$

$$\begin{array}{r} \text{T U} \\ 17 \\ -12 \\ \hline 5 \end{array}$$

$$\begin{array}{r} \text{T U} \\ 32 \\ -11 \\ \hline 21 \end{array}$$

**Check your progress subtraction** -try these calculations:

$\begin{array}{r} 6 \\ -2 \\ \hline ? \end{array}$	$\begin{array}{r} 5 \\ -3 \\ \hline ? \end{array}$	$\begin{array}{r} TU \\ 15 \\ -3 \\ \hline ?? \end{array}$	$\begin{array}{r} TU \\ 25 \\ -12 \\ \hline ?? \end{array}$
--	--	--	---

**Answers subtraction** - as below:

$\begin{array}{r} 6 \\ -2 \\ \hline 4 \end{array}$	$\begin{array}{r} 5 \\ -3 \\ \hline 2 \end{array}$	$\begin{array}{r} 15 \\ -3 \\ \hline 12 \end{array}$	$\begin{array}{r} 25 \\ -12 \\ \hline 13 \end{array}$
--	--	--	---

**Short multiplication** calculations can be written as below. If we have 10 or more on the top line we multiply the Units first (U) and the Tens last (T):

$\begin{array}{r} 3 \\ \times 2 \\ \hline 6 \end{array}$	$\begin{array}{r} 4 \\ \times 3 \\ \hline 12 \end{array}$	$\begin{array}{r} TU \\ 14 \\ \times 2 \\ \hline 28 \end{array}$	$\begin{array}{r} TU \\ 12 \\ \times 4 \\ \hline 48 \end{array}$
--	---	--	--

**Check your progress multiplication** -try these calculations:

$\begin{array}{r} 5 \\ \times 2 \\ \hline ?? \end{array}$	$\begin{array}{r} 12 \\ \times 3 \\ \hline ?? \end{array}$	$\begin{array}{r} TU \\ 24 \\ \times 2 \\ \hline ?? \end{array}$	$\begin{array}{r} TU \\ 33 \\ \times 3 \\ \hline ?? \end{array}$
---	--	--	--

**Answers multiplication** - as below:

$\begin{array}{r} 5 \\ \times 2 \\ \hline 10 \end{array}$	$\begin{array}{r} 12 \\ \times 3 \\ \hline 36 \end{array}$	$\begin{array}{r} 24 \\ \times 2 \\ \hline 48 \end{array}$	$\begin{array}{r} 33 \\ \times 3 \\ \hline 99 \end{array}$
---	--	--	--

**Short division** calculations are written differently. The divisor (how many to share with) is written on the left while the quantity to share is boxed off to the right. The answer is written above the division box (in blue above).

$2 \overline{)6} \quad \text{3}$	$3 \overline{)12} \quad \text{4}$	$2 \overline{)28} \quad \text{14}$	$4 \overline{)48} \quad \text{12}$
$5 \overline{)16} \quad \text{3 remainder 1}$	$10 \overline{)12} \quad \text{1 remainder 2}$	$7 \overline{)39} \quad \text{5 remainder 4}$	$50 \overline{)100} \quad \text{2}$

If the quantity does not share equally we write down a remainder (in red above)

**Check your progress division** -try these calculations:

$$2 \overline{)8} \quad ?$$

$$3 \overline{)21} \quad ?$$

$$4 \overline{)28} \quad ?$$

$$5 \overline{)55} \quad ??$$

$$6 \overline{)25} \quad \text{remainder ?}$$

$$7 \overline{)52} \quad \text{remainder ?}$$

$$8 \overline{)84} \quad \text{remainder ?}$$

$$52 \overline{)104} \quad ?$$

**Answers division** - as below:

$$2 \overline{)8} \quad 4$$

$$3 \overline{)21} \quad 7$$

$$4 \overline{)28} \quad 7$$

$$5 \overline{)55} \quad 11$$

$$6 \overline{)25} \quad \text{remainder 1}$$

$$7 \overline{)52} \quad \text{remainder 3}$$

$$8 \overline{)84} \quad \text{remainder 4}$$

$$52 \overline{)104} \quad 2$$

### Using Long calculations - addition, subtraction, multiplication and division

**Long addition** calculations - With larger numbers the Unit columns can add up to ten or more. To solve this kind of problem we first add the Unit column (blue) but in the answer box only write the unit part of the answer. We *carry over* the ten value to the Ten's column, then sum that column with the carried over as the full answer.

In *example one*, the step one adds the units column  $5 + 7 = 12$ . We write the 2 from the 12 in our answer box and then pencil in 1 (shown red) next to column of tens. We then add the new tens column 1 ten + 2 tens + the carried over ten and find the result is 4 lots of ten. The solution is 42. For larger sums we may need a step three with a Hundreds column (H).

**Example :**

Step 1

$$\begin{array}{r} \text{T U} \\ 1 \text{ } 5 \\ +2 \text{ } 7 \\ \hline ? \text{ } 2 \end{array}$$

Step 2

$$\begin{array}{r} \text{T U} \\ \textcolor{red}{1} 1 \text{ } 5 \\ +2 \text{ } 7 \\ \hline 4 \text{ } 2 \end{array}$$

**Example :**

Step 1

$$\begin{array}{r} \text{T U} \\ 3 \text{ } 5 \\ +4 \text{ } 9 \\ \hline ? \text{ } 4 \end{array}$$

Step 2

$$\begin{array}{r} \text{T U} \\ \textcolor{red}{1} 3 \text{ } 5 \\ +4 \text{ } 9 \\ \hline 8 \text{ } 4 \end{array}$$

**Example :**

Step 1

$$\begin{array}{r} \text{H T U} \\ 2 \text{ } 1 \text{ } 5 \\ +4 \text{ } 2 \text{ } 7 \\ \hline ? \text{ } 3 \text{ } 2 \end{array}$$

Step 2 & 3

$$\begin{array}{r} \text{H T U} \\ 2 \textcolor{red}{1} 1 \text{ } 5 \\ +4 \text{ } 2 \text{ } 7 \\ \hline 6 \text{ } 4 \text{ } 2 \end{array}$$

**Example :**

Step 1

$$\begin{array}{r} \text{H T U} \\ 3 \text{ } 6 \text{ } 5 \\ +5 \text{ } 8 \text{ } 9 \\ \hline ? \text{ } ? \text{ } ? \text{ } 4 \end{array}$$

Step 2 & 3

$$\begin{array}{r} \text{H T U} \\ \textcolor{red}{1} 4 \textcolor{red}{1} 6 \text{ } 5 \\ +5 \text{ } 8 \text{ } 9 \\ \hline 1 \text{ } 0 \text{ } 5 \text{ } 4 \end{array}$$



**Check your progress with long addition sums** - try these calculations:

**Question 1:**

Step 1

$$\begin{array}{r} \text{T U} \\ 56 \\ +39 \\ \hline ?? \\ \hline \end{array}$$

**Question 2:**

Step 1

$$\begin{array}{r} \text{T U} \\ 78 \\ +59 \\ \hline ?? \\ \hline \end{array}$$

**Question 3:**

Step 1

$$\begin{array}{r} \text{H T U} \\ 635 \\ +267 \\ \hline ??? \\ \hline \end{array}$$

**Question 4:**

Step 1

$$\begin{array}{r} \text{H T U} \\ 465 \\ +589 \\ \hline ??? \\ \hline \end{array}$$

**Answers with long addition sums** - as below: - try these calculations:

**Answer 1:**

Step 1

$$\begin{array}{r} \text{T U} \\ 56 \\ +39 \\ \hline ?5 \\ \hline \end{array}$$

Step 2

$$\begin{array}{r} \text{T U} \\ \textcolor{red}{1}56 \\ +39 \\ \hline 95 \\ \hline \end{array}$$

**Answer 2:**

Step 1

$$\begin{array}{r} \text{T U} \\ 78 \\ +59 \\ \hline ?7 \\ \hline \end{array}$$

Step 2

$$\begin{array}{r} \text{T U} \\ \textcolor{red}{1}78 \\ +59 \\ \hline 137 \\ \hline \end{array}$$

**Answer 3:**

Step 1

$$\begin{array}{r} \text{H T U} \\ 635 \\ +267 \\ \hline ??? \\ \hline \end{array}$$

Step 2 & 3

$$\begin{array}{r} \text{H T U} \\ \textcolor{red}{1}6\textcolor{red}{3}5 \\ +267 \\ \hline 902 \\ \hline \end{array}$$

**Answer 4:**

Step 1

$$\begin{array}{r} \text{H T U} \\ 465 \\ +589 \\ \hline ???4 \\ \hline \end{array}$$

Step 2 & 3

$$\begin{array}{r} \text{H T U} \\ \textcolor{red}{1}4\textcolor{red}{6}5 \\ +589 \\ \hline 1054 \\ \hline \end{array}$$

**Long subtraction** calculations – either of two methods described below are used:

**First method** - When the top line units are greater than the bottom line, we have to *borrow* from the next greater column (tens or hundreds). The first Step *borrows* from the ten column by pencilling **1** (ten) next in the unit's column enabling completion of the units part of the answer. To pay back the borrowed ten we cross out the bottom ten's numeral and add to it a ten payback correction.

In *example nine*, subtracting the units 5 - 6 not possible, so the first Step is to borrow as  $\textcolor{red}{1}5 - 6 = 9$ . But the ten has to be paid back so in Step two the 2 (tens) bottom numeral is crossed and replaced with 3, so enabling the solution of 19 to be written in the answer box. In *example eleven* we have to borrow from both the ten's and from the hundred's column to get our answer.

Example :

$$\begin{array}{r} \text{T U} \\ 4 \text{ } ^1 5 \\ - 2 \text{ } 6 \\ \hline ? \text{ } 9 \end{array}$$

$$\begin{array}{r} \text{T U} \\ 4 \text{ } ^1 5 \\ - \textcolor{red}{3} \textcolor{blue}{2} \text{ } 6 \\ \hline 1 \text{ } 9 \end{array}$$

Example :

$$\begin{array}{r} \text{T U} \\ 7 \text{ } ^1 6 \\ - 4 \text{ } 9 \\ \hline ? \text{ } 7 \end{array}$$

$$\begin{array}{r} \text{T U} \\ 7 \text{ } ^1 6 \\ - \textcolor{red}{5} \textcolor{blue}{4} \text{ } 9 \\ \hline 2 \text{ } 7 \end{array}$$

Example :

$$\begin{array}{r} \text{H T U} \\ 7 \text{ } 2 \text{ } ^1 5 \\ - 4 \text{ } 2 \text{ } 8 \\ \hline ? \text{ } ? \text{ } 7 \end{array}$$

$$\begin{array}{r} \text{H T U} \\ 7 \text{ } ^1 2 \text{ } ^1 5 \\ - 4 \textcolor{red}{3} \textcolor{blue}{2} \text{ } 8 \\ \hline 2 \text{ } 9 \text{ } 7 \end{array}$$

$$\begin{array}{r} \text{H T U} \\ 7 \text{ } ^1 2 \text{ } ^1 5 \\ - \textcolor{red}{5} \textcolor{blue}{4} \textcolor{red}{3} \textcolor{blue}{2} \text{ } 8 \\ \hline 2 \text{ } 9 \text{ } 7 \end{array}$$

Example :

$$\begin{array}{r} \text{H T U} \\ 6 \text{ } 6 \text{ } ^1 7 \\ - 5 \text{ } 8 \text{ } 9 \\ \hline ? \text{ } ? \text{ } 8 \end{array}$$

$$\begin{array}{r} \text{H T U} \\ 6 \text{ } ^1 6 \text{ } ^1 7 \\ - 5 \textcolor{red}{9} \textcolor{blue}{8} \text{ } 9 \\ \hline ? \text{ } 7 \text{ } 8 \end{array}$$

$$\begin{array}{r} \text{H T U} \\ 6 \text{ } ^1 6 \text{ } ^1 7 \\ - \textcolor{red}{6} \textcolor{red}{9} \textcolor{blue}{8} \text{ } 9 \\ \hline 0 \text{ } 7 \text{ } 8 \end{array}$$

Note in *example twelve* the hundreds column of 6 hundreds minus 6 hundreds gives an answer of zero hundreds (0). If the taking away number is larger than the number to be taken away from then the arrangement above will not work. In such cases we reverse position of the lines so we can continue taking away a smaller number from a larger, but call the answer we call a minus number.

See *example thirteen* where we have  $637 - 834 = -197$ . For our calculation to work we must take away 637 (the smaller) from 834 (the larger) and call the answer a minus number (-197)

Example :

$$\begin{array}{r} \text{H T U} \\ - 8 \text{ } 3 \text{ } ^1 4 \\ 6 \text{ } 3 \text{ } 7 \\ \hline ? \text{ } ? \text{ } 7 \end{array}$$

$$\begin{array}{r} \text{H T U} \\ - 8 \text{ } ^1 3 \text{ } ^1 4 \\ 6 \textcolor{red}{4} \textcolor{blue}{3} \text{ } 7 \\ \hline ? \text{ } 9 \text{ } 7 \end{array}$$

$$\begin{array}{r} \text{H T U} \\ - 8 \text{ } ^1 3 \text{ } ^1 4 \\ \textcolor{red}{7} \textcolor{red}{6} \textcolor{red}{4} \textcolor{blue}{3} \text{ } 7 \\ \hline - 1 \text{ } 9 \text{ } 7 \end{array}$$

**Long subtraction** calculations

**Second method** - When the top line units are greater than the bottom line, we have to *borrow* from a higher column. The first Step *borrow*s from the ten's column by pencilling 1 (ten) next in the top line unit so enabling completion of the units part of the answer. To pay back the borrowed ten we cross out top line ten's numeral and subtract a ten from it for a payback correction.

In *example fourteen*, Step One finds subtracting the units 5 - 6 not possible, so ten is borrowed to calculate  $15 - 6 = 9$ . But the ten has to be paid back so in Step Two the 4 (tens) top numeral is crossed out and replaced with ten less (3), so enabling the answer box to be completed with the same solution as the first method of 19. In *example sixteen* we have to borrow from the ten's and from the hundred's column to get our answer.

**Example :**

Step 1

	T	U
	4	5
-	2	6
	?	9

Step 2

	T	U
	<del>4</del> <sup>3</sup>	5
-	2	6
	1	9

**Example :**

Step 1

	T	U
	7	6
-	4	9
	?	7

Step 2

	T	U
	<del>7</del> <sup>6</sup>	6
-	4	9
	2	7

**Example :**

Step 1

	H	T	U
	7	2	5
-	4	2	8
	?	?	7

Step 2

	H	T	U
	7	<del>2</del> <sup>1</sup>	5
-	4	2	8
	2	9	7

Step 3

	H	T	U
	<del>6</del> <sup>5</sup>	<del>7</del> <sup>1</sup>	5
-	4	2	8
	2	9	7

**Example :**

Step 1

	H	T	U
	6	6	7
-	5	8	9
	?	?	8

Step 2

	H	T	U
	<del>6</del> <sup>5</sup>	<del>6</del> <sup>5</sup>	7
-	5	8	9
	?	7	8

Step 3

	H	T	U
	<del>5</del> <sup>4</sup>	<del>5</del> <sup>4</sup>	7
-	5	8	9
	0	7	8

**Check your progress with long subtraction** - try these calculations:

**Question 1:**

	T	U
	7	7
-	5	8
	?	?

**Question 2:**

	T	U
	4	2
-	3	9
	?	?

**Question 3:**

	H	T	U
	6	2	3
-	4	8	9
	?	?	?

**Question 4:**

	H	T	U
	8	5	3
-	5	6	5
	?	?	?

### Answers long subtraction:

#### Answer 1:

$$\begin{array}{r} \text{Step 1} \\ \text{T U} \\ 7 \text{ } ^1 7 \\ - 5 \text{ } 8 \\ \hline \text{? } 9 \end{array}$$

$$\begin{array}{r} \text{Step 2} \\ \text{T U} \\ 7 \text{ } ^1 7 \\ - \cancel{6} 5 \text{ } 8 \\ \hline 1 \text{ } 9 \end{array}$$

#### Answer 2:

$$\begin{array}{r} \text{Step 1} \\ \text{T U} \\ 4 \text{ } ^1 2 \\ - 3 \text{ } 9 \\ \hline \text{? } 3 \end{array}$$

$$\begin{array}{r} \text{Step 2} \\ \text{T U} \\ 4 \text{ } ^1 2 \\ - \cancel{4} 3 \text{ } 9 \\ \hline 0 \text{ } 3 \end{array}$$

#### Answer 3:

Step 1

$$\begin{array}{r} \text{H T U} \\ 6 \text{ } 2 \text{ } ^1 3 \\ - 4 \text{ } 8 \text{ } 9 \\ \hline \text{? } \text{? } 4 \end{array}$$

Step 2

$$\begin{array}{r} \text{H T U} \\ 6 \text{ } ^1 2 \text{ } ^1 3 \\ - \cancel{4} 9 \text{ } 8 \text{ } 9 \\ \hline \text{? } 3 \text{ } 4 \end{array}$$

Step 3

$$\begin{array}{r} \text{H T U} \\ 6 \text{ } ^1 2 \text{ } ^1 3 \\ - \cancel{5} \cancel{4} 9 \text{ } 8 \text{ } 9 \\ \hline 1 \text{ } 3 \text{ } 4 \end{array}$$

#### Answer 4:

Step 1

$$\begin{array}{r} \text{H T U} \\ 8 \text{ } 5 \text{ } ^1 3 \\ - 5 \text{ } 6 \text{ } 5 \\ \hline \text{? } \text{? } 8 \end{array}$$

Step 2

$$\begin{array}{r} \text{H T U} \\ 8 \text{ } ^1 5 \text{ } ^1 3 \\ - \cancel{5} 7 \text{ } 6 \text{ } 5 \\ \hline \text{? } 8 \text{ } 8 \end{array}$$

Step 3

$$\begin{array}{r} \text{H T U} \\ 8 \text{ } ^1 5 \text{ } ^1 3 \\ - \cancel{6} \cancel{5} 7 \text{ } 6 \text{ } 5 \\ \hline 2 \text{ } 8 \text{ } 8 \end{array}$$

**Long multiplication** calculations – Times tables are not readily available for multipliers of over twelve, so greater calculations are done in steps. The first Step is to multiply the units, then the tens and then hundreds. Finally all the answer lines are added. Remember to add a zero (0) as will be required for the ten line to be the correct value, as in *Example 18* below:

#### Example :

Step 1

$$\begin{array}{r} \text{H T U} \\ 2 \text{ } 3 \text{ } 4 \\ \times 1 \text{ } 2 \\ \hline 4 \text{ } 6 \text{ } 8 \end{array}$$

Step 2

$$\begin{array}{r} \text{H T U} \\ 2 \text{ } 3 \text{ } 4 \\ \times 1 \text{ } 2 \\ \hline 4 \text{ } 6 \text{ } 8 \\ 0 \end{array}$$

Step 3

$$\begin{array}{r} \text{H T U} \\ 2 \text{ } 3 \text{ } 4 \\ \times 1 \text{ } 2 \\ \hline 4 \text{ } 6 \text{ } 8 \\ \underline{2 \text{ } 3 \text{ } 4 \text{ } 0} \end{array}$$

Step 4

$$\begin{array}{r} \text{H T U} \\ 2 \text{ } 3 \text{ } 4 \\ \times 1 \text{ } 2 \\ \hline 4 \text{ } 6 \text{ } 8 \\ + 2 \text{ } 3 \text{ } 4 \text{ } 0 \\ \hline \underline{2 \text{ } 8 \text{ } 0 \text{ } 8} \end{array}$$

**Example :**

Step 1	Step 2	Step 3	Step 4
$  \begin{array}{r}  \text{H T U} \\  3563 \\  \times 235 \\  \hline  2815  \end{array}  $	$  \begin{array}{r}  \text{H T U} \\  1 \\  3563 \\  \times 235 \\  \hline  2815 \\  16890  \end{array}  $	$  \begin{array}{r}  \text{H T U} \\  1 \\  3563 \\  \times 235 \\  \hline  2815 \\  16890 \\  112600  \end{array}  $	$  \begin{array}{r}  \text{T H T U} \\  1 \\  3563 \\  \times 235 \\  \hline  2815 \\  16890 \\  112600 \\  +112600 \\  \hline  13,2305  \end{array}  $

**Check your progress multiplication** -try these calculations:

**Question 1:**

$$\begin{array}{r}
 423 \\
 \times 25 \\
 \hline
 ???
 \end{array}$$

**Question 2:**

$$\begin{array}{r}
 237 \\
 \times 52 \\
 \hline
 ???
 \end{array}$$

**Question 3:**

$$\begin{array}{r}
 568 \\
 \times 312 \\
 \hline
 ???
 \end{array}$$

**Question 4:**

$$\begin{array}{r}
 777 \\
 \times 436 \\
 \hline
 ???
 \end{array}$$

**Answers multiplication** - as below:

**Answer 1:**

$$\begin{array}{r}
 423 \\
 \times 25 \\
 \hline
 2115 \\
 8460 \\
 \hline
 10,575
 \end{array}$$

**Answer 2:**

$$\begin{array}{r}
 237 \\
 \times 52 \\
 \hline
 474 \\
 11850 \\
 \hline
 12,324
 \end{array}$$

**Answer 3:**

$$\begin{array}{r}
 568 \\
 \times 312 \\
 \hline
 1136 \\
 5680 \\
 170400 \\
 \hline
 177,216
 \end{array}$$

**Answer 4:**

$$\begin{array}{r}
 777 \\
 \times 436 \\
 \hline
 4662 \\
 23310 \\
 310800 \\
 \hline
 338,772
 \end{array}$$

**Long division** calculations – In example 20 below, Step 1 estimates how many divisors (5) fit in the left hand boxed numeral (1). If it will not fit, Step 2 estimates how many fit into the first + the second numerals (1 & 3), in the example giving 2. As  $2 \times 5 = 10$  we subtract 10 from 1 & 3 to leave 3. Step 3 drops the third numeral (5) to give 35. Step 4 finds that the divisor fits 7 times into 35 giving a final answer of 27.

**Example :**

Step 1	Step 2	Step 3	Step 4
$  \begin{array}{r}  022 \\  5 \overline{)135}  \end{array}  $	$  \begin{array}{r}  22 \\  5 \overline{)135} \\  -10 \\  \hline  3  \end{array}  $	$  \begin{array}{r}  22 \\  5 \overline{)135} \\  -10 \\  \hline  35  \end{array}  $	$  \begin{array}{r}  27 \\  5 \overline{)135} \\  -10 \\  \hline  35  \end{array}  $

Example :

Step 1

$$\begin{array}{r} 1?? \\ 8 \overline{)968} \\ \underline{-8} \phantom{00} \\ \phantom{00} \end{array}$$

Step 2

$$\begin{array}{r} 1?? \\ 8 \overline{)968} \\ \underline{-8} \phantom{00} \\ \phantom{00}1 \phantom{00} \end{array}$$

Step 3

$$\begin{array}{r} 12? \\ 8 \overline{)968} \\ \underline{-8} \phantom{00} \\ \phantom{00}16 \phantom{00} \end{array}$$

Step 4

$$\begin{array}{r} 121 \\ 8 \overline{)968} \\ \underline{-8} \phantom{00} \\ \phantom{00}16 \phantom{00} \\ \underline{-16} \phantom{00} \\ \phantom{00}08 \phantom{00} \end{array}$$

Example :

Step 1

$$\begin{array}{r} 2?? \\ 14 \overline{)2968} \\ \underline{-28} \phantom{00} \\ \phantom{00} \end{array}$$

Step 2

$$\begin{array}{r} 21? \\ 14 \overline{)2968} \\ \underline{-28} \phantom{00} \\ \phantom{00}16 \phantom{00} \end{array}$$

Step 3

$$\begin{array}{r} 0212 \\ 14 \overline{)2968} \\ \underline{-28} \phantom{00} \\ \phantom{00}16 \phantom{00} \\ \underline{-14} \phantom{00} \\ \phantom{00}28 \phantom{00} \end{array}$$

Step 4

$$\begin{array}{r} 0212 \\ 14 \overline{)2968} \\ \underline{-28} \phantom{00} \\ \phantom{00}16 \phantom{00} \\ \underline{-14} \phantom{00} \\ \phantom{00}28 \phantom{00} \end{array}$$

Example :

Step 1

$$\begin{array}{r} 1??? \\ 35 \overline{)51415} \\ \underline{-35} \phantom{00} \\ \phantom{00} \end{array}$$

Step 2

$$\begin{array}{r} 14?? \\ 35 \overline{)51415} \\ \underline{-35} \phantom{00} \\ \phantom{00}164 \phantom{00} \end{array}$$

Step 3

$$\begin{array}{r} 146?? \\ 35 \overline{)51415} \\ \underline{-35} \phantom{00} \\ \phantom{00}164 \phantom{00} \\ \underline{-140} \phantom{00} \\ \phantom{00}241 \phantom{00} \end{array}$$

Step 4

$$\begin{array}{r} 1469 \\ 35 \overline{)51415} \\ \underline{-35} \phantom{00} \\ \phantom{00}164 \phantom{00} \\ \underline{-140} \phantom{00} \\ \phantom{00}241 \phantom{00} \\ \underline{-210} \phantom{00} \\ \phantom{00}315 \phantom{00} \end{array}$$

Example :

If the quantity does not share equally we write down a remainder (in red below), or write it as a fraction (see next Section 2.3)

Step 1

$$\begin{array}{r} 2??? \\ 21 \overline{)53415} \\ \underline{-42} \phantom{00} \\ \phantom{00} \end{array}$$

Step 2

$$\begin{array}{r} 25?? \\ 21 \overline{)53415} \\ \underline{-42} \phantom{00} \\ \phantom{00}114 \phantom{00} \end{array}$$

Step 3

$$\begin{array}{r} 254? \\ 21 \overline{)53415} \\ \underline{-42} \phantom{00} \\ \phantom{00}114 \phantom{00} \\ \underline{-105} \phantom{00} \\ \phantom{00}91 \phantom{00} \end{array}$$

Step 4

$$\begin{array}{r} 2543 \\ 21 \overline{)53415} \\ \underline{-42} \phantom{00} \\ \phantom{00}114 \phantom{00} \\ \underline{-105} \phantom{00} \\ \phantom{00}91 \phantom{00} \\ \underline{-84} \phantom{00} \\ \phantom{00}75 \phantom{00} \\ \underline{-63} \phantom{00} \\ \phantom{00}12 \text{ remains} \end{array}$$



*Check your progress long division* -try these calculations:

Question 1:

$$\begin{array}{r} \text{???} \\ 4 \overline{)624} \end{array}$$

Question 2:

$$\begin{array}{r} \text{????} \\ 12 \overline{)2808} \end{array}$$

Question 3:

$$\begin{array}{r} \text{????} \\ 24 \overline{)2961} \end{array}$$

Question 4:

$$\begin{array}{r} \text{?????} \\ 32 \overline{)81927} \end{array}$$

*Answers long division* - as below:

Answer 1:

$$\begin{array}{r} 156 \\ 4 \overline{)624} \\ \underline{-4} \phantom{00} \\ 22 \phantom{0} \\ \underline{-20} \phantom{0} \\ 24 \end{array}$$

Answer 2:

$$\begin{array}{r} 234 \\ 12 \overline{)2808} \\ \underline{-24} \phantom{00} \\ 40 \phantom{0} \\ \underline{-36} \phantom{0} \\ 48 \end{array}$$

Answer 3:

$$\begin{array}{r} 123 \\ 24 \overline{)2961} \\ \underline{-24} \phantom{00} \\ 56 \phantom{0} \\ \underline{-48} \phantom{0} \\ 81 \phantom{0} \\ \underline{-72} \phantom{0} \\ 9 \text{ remains} \end{array}$$

Answer 4:

$$\begin{array}{r} 2561 \\ 32 \overline{)81927} \\ \underline{-64} \phantom{00} \\ 179 \phantom{0} \\ \underline{-160} \phantom{0} \\ 195 \phantom{0} \\ \underline{-192} \phantom{0} \\ 37 \phantom{0} \\ \underline{-32} \phantom{0} \\ 5 \text{ remains} \end{array}$$

## 2.3 Fractions and decimals

### Fractions

To share a cake we slice it into parts. Each part can be called a *fraction* of the whole cake, for example, slice number 1 below is from a cake with 8 slices. It is one eighth of the cake. All fractions are parts of a whole.



Fraction's are *expressed* (written) separated by a line – as above  $\frac{1}{8}$  of a cake.

A **numerator** - a number on top showing us the parts of the whole we have.  
A **denominator** - a bottom number showing how many parts make up the whole.

The cake could have been cut into any other fractions, such as:

$\frac{1}{2}$  = one slice of a whole cake cut into two = half a cake

$\frac{1}{4}$  = one slice of a whole cake cut into four = quarter of a cake

$\frac{1}{10}$  = one slice of a whole cake cut into ten = tenth of a cake

### Calculating with fractions

#### ***Addition – with a common denominator***

To add fractions the denominators (the number below the line) must be the same (called *common*). The answer is found by totalling the numerators (number above the line) of each fraction and expressing with the common denominator, as below:

$$\frac{1}{20} + \frac{2}{20} + \frac{3}{20} + \frac{4}{20} = \frac{10}{20} \quad \text{half a cake}$$

### Expression and Substitution

However, we could have received several cake slices expressed as fractions below:

$$\frac{4}{8} = \text{four slices of a whole cake cut into eight} = \frac{1}{2} \quad \text{half of cake}$$

$$\frac{4}{12} = \text{four slices of a whole cake cut into twelve} = \frac{1}{3} \quad \text{third of cake}$$

You will notice that both 4 slices of an 8 slice cake and 10 slices of a 20 slice cake both are half of each cake. So fractions can be expressed with differing numerator and denominator and still be the same value as a part of the whole, for instance:

$$\frac{10}{20} = \frac{1}{2} = \frac{5}{10} = \frac{2}{4} = \frac{10}{20}$$

The rule is that we must change both the top and the bottom line of a fraction with the same factor (divisor or multiplier) if the fraction is to retain its value. Applying this rule is called *substitution*.

$$\frac{10}{20} \begin{matrix} \times 4 \\ \times 4 \end{matrix} = \frac{40}{80} \begin{matrix} \div 10 \\ \div 10 \end{matrix} = \frac{4}{8} \begin{matrix} \times 3 \\ \times 3 \end{matrix} = \frac{12}{24} \begin{matrix} \div 12 \\ \div 12 \end{matrix} = \frac{1}{2} \quad \checkmark$$

Note below that substitution does not work by adding or subtracting to numerator and denominator. Only by multiplying and dividing equally gives a correct answer.

$$\frac{10}{20} \begin{matrix} + 4 \\ + 4 \end{matrix} = \frac{14}{24} \quad \text{X} \quad \text{Wrong!!} \qquad \frac{10}{20} \begin{matrix} - 4 \\ - 4 \end{matrix} = \frac{6}{16} \quad \text{X} \quad \text{Wrong !!}$$

### Addition – fractions without a common denominator

We need a *common denominator* to add fractions. But in the example below different cakes have been cut into different numbers of slices. Each cake's denominator is different. We have to change each fraction to the same denominator but still express its same value (part of a whole cake). All fractions must be of same sized slices.

To find our *common denominator* we could multiply all denominators, as below:

$$\begin{array}{ccccccc} \frac{10}{20} & + & \frac{3}{5} & + & \frac{6}{10} & + & \frac{2}{4} & = & \frac{?}{?} \\ 20 & \times & 5 & \times & 10 & \times & 4 & = & 4000 \end{array}$$

The common denominator in this case is 4000, a very large number of tiny slices of a cake and a complex number for calculations. It will be easier for us to find the *lowest common denominator*, the smallest number that all denominators will divide into without leaving any remainder. In the example below it is **20**.

Step 1 - draw a long line above our chosen lowest common denominator **20**.

Step 2 - divide the lowest common denominator with each fraction's denominator.

Step 3 - multiply the answer with each old numerator. Mathematical balance is regained with the value being equal to each old fraction's value. Finally total the numerators and express the answer with the lowest common denominator.

$$\frac{10}{20} + \frac{4}{5} + \frac{6}{10} + \frac{2}{4} = \frac{?}{?}$$

Step 3:

$$1 \times 10 = 20 \quad 4 \times 4 = 12 \quad 2 \times 6 = 12 \quad 5 \times 2 = 10$$

Step 2:

$$20 \div 20 = 1 \quad 20 \div 5 = 4 \quad 20 \div 10 = 2 \quad 20 \div 4 = 5$$

Step 1:

$$\frac{10 + 16 + 12 + 10}{20} = \frac{48}{20} = 2 \frac{8}{20} = 2 \frac{2}{5}$$

Unlike earlier examples our answer is an *improper fraction*. The numerator 48 is greater than the denominator 20. Remember, the slices are from more than one cake. To express properly we divide the numerator with the denominator. The 48 pieces from 20 sliced cakes could be reassembled as 2 wholes with 8 slices left over. Eight twentieths can be *substituted* as the same as two fifths of a cake.

Sometimes we have to add whole numbers with fractions as below. The method is to convert each whole number with fraction into an improper fraction, find the common denominator and then continue as normal addition. The answer, an improper fraction, is converted back to whole number with any remainder fraction. The remainder can be substituted with lowest denominator for the simplest expression.

$$1 \frac{1}{2} + 2 \frac{2}{3} + 1 \frac{1}{6} = ?$$

$$\frac{2}{2} + \frac{1}{2} + \frac{6}{3} + \frac{2}{3} + \frac{6}{6} + \frac{1}{6} = ?$$

$$\frac{3}{2} + \frac{8}{3} + \frac{7}{6} = ?$$

$$\frac{9 + 16 + 7}{6} = \frac{32}{6} = 5 \frac{2}{6} = 5 \frac{1}{3}$$

### Subtraction

As with adding, subtracting fractions requires a common denominator. The answer is found by subtracting the numerators and expressing with the common denominator:

$$\frac{19}{20} - \frac{10}{20} - \frac{5}{20} - \frac{3}{20} = \frac{1}{20}$$

If the denominator is not common, we have to *substitute* to express all with the same denominator.

Step 1 - draw a long line above our chosen lowest common denominator **20**.

Step 2 - divide the lowest common denominator with each fraction's denominator.

Step 3 - multiply the answer with each old numerator. Mathematical balance is regained with the value being equal to each old fraction's value. Finally subtract the numerators and express the answer with the lowest common denominator.

$$\frac{19}{20} - \frac{1}{2} - \frac{2}{10} - \frac{1}{5} = \frac{?}{?}$$

Step 3:

$$1 \times 19 = 19 \quad 10 \times 1 = 10 \quad 2 \times 2 = 4 \quad 4 \times 1 = 4$$

Step 2:

$$20 \div 20 = 1 \quad 20 \div 2 = 10 \quad 20 \div 10 = 2 \quad 20 \div 4 = 5$$

Step 1:

$$\frac{19 - 10 - 4 - 4}{20} = \frac{1}{20}$$

Sometimes we have to subtract whole numbers with fractions. The method is to convert each number with fraction into an improper fraction, find the common denominator and then continue as normal subtraction. The answer, an improper fraction, is converted back to whole number with any remainder. The remainder can be substituted with lowest denominator for the simplest expression.

$$5 \frac{1}{2} - 1 \frac{2}{3} - 2 \frac{3}{6} = ?$$

$$\frac{10}{2} + \frac{1}{2} - \frac{3}{3} + \frac{2}{3} - \frac{12}{6} + \frac{3}{6} = ?$$

$$\frac{11}{2} - \frac{5}{3} - \frac{15}{6} = ?$$

$$\frac{33 - 10 - 15}{6} = \frac{8}{6} = 1 \frac{2}{6} = 1 \frac{1}{3}$$

**Check your progress adding and subtracting fractions** -try these calculations:

**Question 1:**

$$\frac{1}{6} + \frac{2}{6} + \frac{1}{6} = \frac{?}{?}$$

**Question 2:**

$$\frac{2}{10} + \frac{3}{10} + \frac{4}{10} = \frac{?}{?}$$

**Question 3:**

$$\frac{1}{2} + \frac{2}{3} + \frac{5}{6} = \frac{?}{?}$$

**Question 4:**

$$\frac{9}{10} - \frac{2}{10} - \frac{5}{10} = \frac{?}{?}$$

**Question 5:**

$$\frac{19}{22} - \frac{4}{22} - \frac{4}{22} = \frac{?}{?}$$

**Question 6:**

$$\frac{9}{12} - \frac{1}{3} - \frac{2}{6} = \frac{?}{?}$$

**Question 7:**

$$1\frac{1}{2} + 3\frac{3}{4} + 2\frac{3}{8} = \frac{?}{?}$$

**Question 8:**

$$8\frac{2}{3} - 2\frac{1}{2} - 3\frac{4}{6} = \frac{?}{?}$$

**Answers adding and subtracting fractions** - as below:

**Answer 1:**

$$\frac{1}{6} + \frac{2}{6} + \frac{1}{6} = \frac{4}{6}$$

**Answer 2:**

$$\frac{2}{10} + \frac{3}{10} + \frac{4}{10} = \frac{9}{10}$$

**Answer 3:**

$$\frac{1}{2} + \frac{2}{3} + \frac{5}{6} = \frac{3}{6} + \frac{4}{6} + \frac{5}{6} = \frac{12}{6} = 2$$

**Answer 4:**

$$\frac{9}{10} - \frac{2}{10} - \frac{5}{10} = \frac{2}{10}$$

**Answer 5:**

$$\frac{19}{22} - \frac{4}{22} - \frac{4}{22} = \frac{11}{22} = \frac{1}{2}$$

**Answer 6:**

$$\frac{9}{12} - \frac{1}{3} - \frac{2}{6} = \frac{9}{12} - \frac{4}{12} - \frac{4}{12} = \frac{1}{12}$$

**Answer 7:**

$$1\frac{1}{2} + 3\frac{3}{4} + 2\frac{3}{8} = \frac{3}{2} + \frac{15}{4} + \frac{19}{8} = \frac{12 + 30 + 19}{8} = \frac{61}{8} = 7\frac{5}{8}$$

**Answer 8:**

$$8\frac{2}{3} - 2\frac{1}{2} - 3\frac{4}{6} = \frac{26}{3} - \frac{5}{2} - \frac{22}{6} = \frac{52 - 15 - 22}{6} = \frac{15}{6} = 2\frac{3}{6}$$



### Multiplication with fractions

Common denominators are not required to multiply fractions, as shown below:

Example :

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Example :

$$\frac{1}{2} \times \frac{2}{3} \times \frac{4}{6} = \frac{8}{36} = \frac{2}{9}$$

With whole number and fraction combinations, each can be multiplied separately. Note that the answer is usually expressed as the simplest fraction (substituted with a lowest common denominator). In 8 over 36 above, both are divided by 4.

Example :

$$2 \frac{1}{2} \times 3 \frac{3}{4} = 6 \frac{3}{8}$$

Example :

$$3 \frac{2}{3} \times 4 \frac{4}{6} \times 6 \frac{1}{4} = 72 \frac{8}{72} = 72 \frac{1}{9}$$

*Check your progress multiplying fractions* -try these calculations:

Question 1:

$$\frac{3}{5} \times \frac{2}{3} = ?$$

Question 2:

$$\frac{1}{2} \times \frac{5}{6} \times \frac{2}{3} = ?$$

Question 3:

$$2 \frac{5}{6} \times 2 = ?$$

Question 4:

$$5 \frac{2}{3} \times 4 \frac{5}{6} \times 3 \frac{1}{5} = ?$$

*Answers multiplying fractions* - as below:

Answer 1:

$$\frac{3}{5} \times \frac{2}{3} = \frac{6}{15}$$

Answer 2:

$$\frac{1}{2} \times \frac{5}{6} \times \frac{2}{3} = \frac{10}{36} = \frac{5}{18}$$

Answer 3:

$$2 \frac{5}{6} \times 2 = 4 \frac{15}{24}$$

Answer 4:

$$5 \frac{2}{3} \times 4 \frac{5}{6} \times 3 \frac{1}{5} = 60 \frac{10}{90} = 60 \frac{1}{9}$$

### Division with fractions

Dividing fractions with fractions is easiest achieved by inverting the second fraction's numerator and denominator and then multiplying. Note that the answer is usually expressed by substitution as the simplest fraction.

$$\frac{\text{NUM}}{\text{DEN}} \div \frac{\text{num}}{\text{den}} = \frac{\text{NUM}}{\text{DEN}} \times \frac{\text{den}}{\text{num}} = \frac{\text{Nd}}{\text{Dn}}$$

Example :

$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = 1 \frac{2}{4} = 1 \frac{1}{2}$$

Example :

$$\frac{6}{8} \div \frac{1}{3} = \frac{6}{8} \times \frac{3}{1} = \frac{18}{8} = 2 \frac{2}{8} = 2 \frac{1}{4}$$

With whole number and fraction combinations it is easiest to convert each to an improper fraction. Then invert the dividing fraction and multiply with it. Note that the answer is usually expressed as the simplest.

Example :

$$1 \frac{1}{2} \div \frac{1}{4} = \frac{2+1}{2} \div \frac{1}{4} = \frac{3}{2} \div \frac{1}{4} = \frac{3}{2} \times \frac{4}{1} = \frac{12}{2} = 6$$

Example :

$$2 \frac{1}{3} \div 1 \frac{2}{6} = \frac{6+1}{3} \div \frac{6+2}{6} = \frac{7}{3} \div \frac{8}{6} = \frac{7}{3} \times \frac{6}{8} = \frac{42}{24} = 1 \frac{18}{24}$$

*Check your progress dividing fractions* -try these calculations:

Question 1:

$$\frac{1}{2} \div \frac{1}{4} = ?$$

Question 2:

$$\frac{5}{6} \div \frac{2}{3} = ?$$

Question 3:

$$2 \frac{1}{2} \div 1 \frac{1}{4} = ?$$

Question 4:

$$2 \frac{5}{6} \div 1 \frac{6}{9} = ?$$

*Answers dividing fractions* - as below:

Answer 1:

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2$$

Answer 2:

$$\frac{5}{6} \div \frac{2}{3} = \frac{5}{6} \times \frac{3}{2} = \frac{15}{12} = 1 \frac{3}{12} = 1 \frac{1}{4}$$

Answer 3:

$$2 \frac{1}{2} \div 1 \frac{1}{4} = \frac{4+1}{2} \div \frac{4+1}{4} = \frac{5}{2} \div \frac{5}{4} = \frac{5}{2} \times \frac{4}{5} = \frac{20}{10} = 2$$

Answer 4:

$$2 \frac{5}{6} \div 1 \frac{6}{9} = \frac{12+5}{6} \div \frac{9+6}{9} = \frac{17}{6} \div \frac{15}{9} = \frac{17}{6} \times \frac{9}{15} = \frac{153}{90} = 1 \frac{21}{30}$$

## Decimals

A dot or *point* (·) following a numeral is another way to express a *fraction* of its whole. A *decimal* point expresses a fraction that is a division of ten or its multiples when using the base ten numbering system ([see more about base in Section 1.3](#)), for instance:

- 1· 0      One whole only.
- 1· 1      One whole and one tenth of a whole.
- 1· 25     One whole and twenty five hundredths whole ( $\frac{1}{4}$ ).
- 1· 5      One whole and five tenths whole ( $\frac{1}{2}$ ).
- 1· 75     One whole and seventy five hundredths whole ( $\frac{3}{4}$ ).
- 1· 9      One whole and nine tenths of a whole.

The decimal place determines the fractions multiple of ten. If it is placed immediately to the right of a whole number it expresses a tenth ( $1 \times 10$ ), if it is placed two spaces to the right it expresses one hundredth ( $10 \times 10$ ), etc, as below:

- 1· 1      One whole and one tenth of a whole.
- 1· 01     One whole and one hundredth of a whole.
- 1· 001    One whole and one thousandths of a whole.
- 1· 0001   One whole and the thousandths of a whole.
- 1· 00001   One whole and one hundred thousandths of a whole.
- 1· 000001 One whole and one millionths of a whole.

### ***Adding and subtracting***

Writing sums in a table format (or columns) ensures that the decimal place of the answer is correctly positioned above the decimal place of the question.

Example:

$$\begin{array}{r} 1\cdot 1 \\ + 3\cdot 6 \\ \hline 4\cdot 7 \end{array}$$

Example:

$$\begin{array}{r} 1\cdot 52 \\ + 33\cdot 34 \\ \hline 34\cdot 86 \end{array}$$

Example:

$$\begin{array}{r} 106\cdot 325 \\ + 521\cdot 520 \\ \hline 627\cdot 845 \end{array}$$

Example:

$$\begin{array}{r} 3\cdot 6 \\ - 1\cdot 1 \\ \hline 2\cdot 5 \end{array}$$

Example:

$$\begin{array}{r} 33\cdot 34 \\ - 1\cdot 52 \\ \hline 31\cdot 82 \end{array}$$

Example:

$$\begin{array}{r} 521\cdot 520 \\ - 106\cdot 325 \\ \hline 415\cdot 195 \end{array}$$

### ***Multiplying***

It is easiest to multiply each whole numeral and decimal numeral as in long multiplication. After completing the calculation, the answer's decimal place is determined from counting the places found in the question, as below:

Example:

$$\begin{array}{r}
 3.6 \\
 \times 1.1 \\
 \hline
 36 \\
 360 \\
 \hline
 3.96
 \end{array}$$

Example:

$$\begin{array}{r}
 33.34 \\
 \times 2.12 \\
 \hline
 6668 \\
 33340 \\
 \hline
 666800 \\
 706808
 \end{array}$$

Example:

$$\begin{array}{r}
 21.511 \\
 \times 6.325 \\
 \hline
 107555 \\
 430220 \\
 \hline
 6453300 \\
 129066000 \\
 \hline
 136.057075
 \end{array}$$

As a double check, ensure that the answer's decimal place roughly meets the expected units. Above  $3 \times 1$  should roughly be 3 units and  $33 \times 2$  should be 66.

### Dividing

It is easiest to multiply each numeral and decimal by a multiple of ten until they become whole numbers, before calculating as in long division. If both the divisor and the dividend are multiplied by the same amount then the proportions and answer will be correct.

Example:

$$(24 \div 0.2) \times 10 = (240 \div 2) =$$

$$\begin{array}{r}
 120 \\
 2 \overline{)240}
 \end{array}$$

Example:

$$(36 \div 0.06) \times 100 = (3600 \div 6) =$$

$$\begin{array}{r}
 600 \\
 6 \overline{)3600}
 \end{array}$$

Example:

$$4.8 \div 2 \times 10 = (48 \div 20) =$$

$$\begin{array}{r}
 2.4 \\
 20 \overline{)48.0} \\
 \underline{40} \phantom{0} \\
 80
 \end{array}$$

Example:

$$(15.75 \div 0.75) \times 100 = (1575 \div 75) =$$

$$\begin{array}{r}
 21 \\
 75 \overline{)1575} \\
 \underline{150} \phantom{0} \\
 75
 \end{array}$$

In the examples above the numerals and decimals were multiplied to become whole numbers, but we could just have easily moved the decimal places to the right to achieve the same result.

### Converting fractions to decimals

To convert fractions to decimals the numerator is divided by the denominator. Don't always expect a simple decimal answer unless the fraction is a division of ten.

Example:

$$\frac{1}{2} = \frac{0.5}{2 \overline{)1.0}}$$

Example:

$$\frac{3}{4} = \frac{0.75}{4 \overline{)3.00}}
 \begin{array}{r}
 28 \\
 20
 \end{array}$$

Example:

$$\frac{5}{8} = \frac{0.625}{8 \overline{)5.000}}
 \begin{array}{r}
 48 \\
 20 \\
 16 \\
 40
 \end{array}$$

**Check your progress decimals** -try these calculations:

Question 1:

$$\begin{array}{r} 5.2 \\ + 3.6 \\ \hline ? \end{array}$$

Question 4:

$$\begin{array}{r} 5.2 \\ - 3.6 \\ \hline ? \end{array}$$

Question 7:

$$\begin{array}{r} 5.7 \\ \times 2.1 \\ \hline ? \end{array}$$

Question 10:

$$7.2 \div 4 = ?$$

Question 2:

$$\begin{array}{r} 11.23 \\ + 24.62 \\ \hline ? \end{array}$$

Question 5:

$$\begin{array}{r} 24.62 \\ - 1.52 \\ \hline ? \end{array}$$

Question 8:

$$\begin{array}{r} 27.32 \\ \times 1.42 \\ \hline ? \end{array}$$

Question 11:

$$34.25 \div 0.25 = ?$$

Question 3:

$$\begin{array}{r} 345.345 \\ + 123.123 \\ \hline ? \end{array}$$

Question 6:

$$\begin{array}{r} 573.267 \\ - 106.325 \\ \hline ? \end{array}$$

Question 9:

$$\begin{array}{r} 41.22 \\ \times 2.33 \\ \hline ? \end{array}$$

**Answers decimals** - as below:

Answer 1:

$$\begin{array}{r} 5.2 \\ + 3.6 \\ \hline 8.8 \end{array}$$

Answer 4:

$$\begin{array}{r} 5.2 \\ - 3.6 \\ \hline 1.6 \end{array}$$

Answer 7:

$$\begin{array}{r} 5.7 \\ \times 2.1 \\ \hline 57 \\ 1140 \\ \hline 11.97 \end{array}$$

Answer 10:

$$(7.2 \div 4) \times 10 = 72 \div 40 =$$

$$\begin{array}{r} 1.8 \\ 40 \overline{)72.0} \\ \underline{40} \\ 320 \end{array}$$

Answer 2:

$$\begin{array}{r} 11.23 \\ + 24.62 \\ \hline 35.85 \end{array}$$

Answer 5:

$$\begin{array}{r} 24.62 \\ - 1.52 \\ \hline 23.10 \end{array}$$

Answer 8:

$$\begin{array}{r} 27.32 \\ \times 1.42 \\ \hline 5464 \\ 109280 \\ 273200 \\ \hline 38.7944 \end{array}$$

Answer 11:

$$(34.25 \div 0.25) \times 100 = 3425 \div 25 =$$

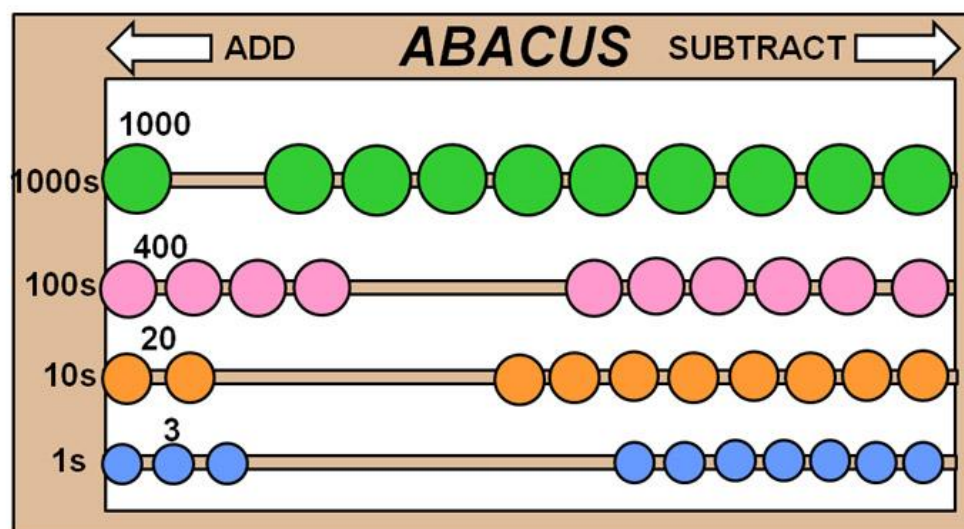
$$\begin{array}{r} 137 \\ 25 \overline{)3425} \\ \underline{25} \\ 925 \end{array}$$

## 2.3 Calculating machines

### The abacus – a counting machine

The abacus is an ancient counting machine that is constructed in a variety of ways across the world. It consists of a frame supporting a number of rails upon which beads can slide, so recording counting operations (addition or subtraction). Each rail supports a group of beads that are attributed with levels of value.

In the decimal based Abacus shown below, the beads on the bottom rail each have a value of one unit, the rail above contains beads of value ten, the rail above contains beads of value one hundred and the rail above contains beads of value one thousand.



When the maximum number of beads for each rail is slid to the left, their combined value will equal one of that above. The operation is to slide one of the above to the left and slide the ten below back to the right. The status shown on the abacus is the value of one thousand, four hundreds, two tens and three units (1423 units in base ten system). This Abacus is limited to recording a maximum value of 10,999 units.

### The calculator

Addition, subtraction, multiplication and division of fractions and decimals, as well as conversions from fraction form to decimal form, can all be undertaken on a standard *calculator*. A *scientific calculator* has additional functions including but not limited to exponents, roots, constants and trigonometry.



The keypads and operation can differ from one calculator to another, as described in the user's manuals, but the keying below is common. If in doubt, as with algebra, use the processing order of *bemdas*.

#### Addition and subtraction

Press all addition values, equal, press all negative values, equal. Example:

$$2 + 3 + 4 + 5 + 6 = 20 \quad 20 - 6 - 5 - 4 - 3 - 2 = 0$$

#### Multiplication and division

Press all multiplication values, equal, press all divisions ( $\div$  key is  $/$ ), equal. Example:

$$2 \times 3 \times 4 \times 5 \times 6 = 720 \quad 720 / 6 / 5 / 4 / 3 / 2 = 0$$

#### Mixed operators

The problem below is set out in the *bemdas* sequence. Example,

$$2 \times 3 \times 4 = 24 \div 2 \div 3 = 4 + 2 + 3 = 9 - 4 - 3 = 2$$

The problem is set out using brackets to clarify the *bemdas* sequence. Example:

$$(2 \times 3 \times 4) = 24 \quad (24 / 2 / 3) = 4 \quad (4 + 2 + 3) = 9 \quad (9 - 4 - 3) = 2$$

#### Fractions

Standard calculators may not have a dedicated fractions keypad. Each fraction must be resolved (as if a division in brackets) into a decimal fraction. Only then can the decimal fractions be re-entered for the division operation. Example:

$$\frac{1}{2} \div \frac{1}{4} =$$

$$(1 / 2) = .5 \quad (1 / 4) = .25 \quad .5 / .25 = 2$$

If a MS (memory store) and a MR (memory recall) are available in your calculator, then rearranging the processing order can simplify keying entries, as below.

$$1 / 4 = .25 \quad \text{MS} \quad 1 / 2 = .5 \quad .5 / \text{MR} = 2$$

Some scientific calculators may have a fraction keypad to press between numerator and denominator entries to enable fractional to be calculated directly. Example:

$$\frac{2}{3} - \left( \frac{4}{6} \times \frac{1}{2} \right) =$$

$$2 \text{ a } \frac{b}{c} 3 - 4 \text{ a } \frac{b}{c} 6 \times 1 \text{ a } \frac{b}{c} 2 = 0.33$$

**Check your progress** - Remember *bemdas* with these calculations:

Question 1.  $12.50 + 7.25 + 2.75 = ?$

Question 2.  $23.12 - 5.70 - 8.22 = ?$

Question 3.  $15.23 \times 3.50 \times 4.23 = ?$

Question 4.  $54.16 \div 0.54 \div 2.34 = ?$

Question 5.  $12.50 \times 7.25 + 2.75 = ?$

Question 6.  $26.12 \div 6.53 - 3.22 = ?$

Question 7.  $15.23 + 3.50 \times 4.23 = ?$

Question 8.  $54.16 - 4.68 \div 2.34 = ?$

Question 9.  $\frac{1}{8} - \left( \frac{3}{4} \times \frac{1}{2} \right) = ?$

Question 10.  $\frac{7}{8} \div \left( \frac{1}{4} \times \frac{9}{8} \right) = ?$

### **Answers**

Answer 1.  $12.5 + 7.25 + 2.75 = 22.50$

Answer 2.  $23.12 - 5.70 - 8.22 = 9.20$

Answer 3.  $15.23 \times 3.50 \times 4.23 = 225.48$

Answer 4.  $54.16 \div 0.54 \div 2.34 = 42.86$

Answer 5.  $12.5 \times 7.25 + 2.75 = 93.375$

Answer 6.  $26.12 \div 6.53 - 3.22 = 0.78$

Answer 7.  $15.23 + 3.50 \times 4.23 = 30.035$

Answer 8.  $54.16 - 4.68 \div 2.34 = 52.16$

Answer 9.  $\frac{1}{8} - \left( \frac{3}{4} \times \frac{1}{2} \right) = -0.25$

Answer 10.  $\frac{7}{8} \div \left( \frac{1}{4} \times \frac{9}{8} \right) = 3.111$

## 2.4 Elementary algebra

### Balanced equations

A number has a value that can also be expressed as calculations equal to that value. These alternative *balanced equations* behave like weighing scales where the same known weight (value) is in balance each side of the pivot point (the equals sign).

<i>Balanced equations</i>										
<u>10</u>	=	<u>5 + 5</u>	=	<u>15 - 5</u>	=	<u>2 x 5</u>	=	<u>50 ÷ 5</u>	=	<u>10</u>
	▲		▲		▲		▲			

If weights are changed equally on each side the scales will still balance. Below, each side of the equations has undergone the operation of multiplication by **2**, the total value increases to **20** but the equations are still balanced. Note that the brackets (*parenthesis*) recommend completing that part first before multiplying by **2**. Also, multiplication can be indicated by placing a multiplying number to the left of the multiplied, for instance **2(5 + 5)** means **2 x (5 + 5)**, and **2a** means **2 x a**.

<i>Operations with equations</i>										
<b>2 x 10</b>	=	<b>2(5 + 5)</b>	=	<b>2(15 - 5)</b>	=	<b>2 (2 x 5)</b>	=	<b>2(50 ÷ 5)</b>	=	<b>20</b>
	▲		▲		▲		▲		▲	

The equations above have *known* values as they were written with numbers. But *algebra* can express a value that is *unknown* (called *variable*) by substituting letters for values. Re-assembling balanced equations using algebra can make calculations easier to understand and simpler to solve, as below:

Balance (*equality*) is shown if *adding* **c** to each side of this equation.

If **a = b** then **a + c = b + c**

It is also shown if *subtracting* **c** from each side of this equation.

If **a + c = b + c** then **a + c - c = b + c - c** then **a = b**

It is also shown if multiplying both sides of this equation by **c**.

If **a = b** then **ac = bc**

It is also shown if dividing both sides of this equation by **c**.

If **ac = bc** then  $\frac{ac}{c} = \frac{bc}{c}$  then **a = b**

A basic rule or truth is that when a value is multiplied by 1 it is not changed and when a value is multiplied by its opposite (its reciprocal) the result is 1.

$\times 1$					$\times \text{reciprocal}$				
$b$	$\times$	$1$	$=$	$b$	$b$	$\times$	$\frac{1}{b}$	$=$	$1$
or									
$y$	$\times$	$1$	$=$	$y$	$y$	$\times$	$\frac{1}{y}$	$=$	$1$

Note above that some operations are paired, in that they can reverse the operation of the other. Above, the multiplication by a reciprocal has the same result as division. This property was used earlier when simplify fraction operations by the process of *substitution*. This pairing is seen with the operations:

<i>Paired operations</i>				
$+$	$\leftarrow$	add and subtract	$\rightarrow$	$-$
$\times$	$\leftarrow$	multiply and divide	$\rightarrow$	$\div$
$a^2$	$\leftarrow$	power or exponents and roots	$\rightarrow$	$\sqrt{\phantom{x}}$
(see section 1.4)				

## Algebraic notation

Names are given to the parts within an algebraic expression. This assists in when describing algebra and when deciding the order (steps) when calculating.

<i>Algebraic notation</i>									
$1$	$2$	$3$		$4$		$1$	$2$	$2$	
$4$	$z$	$^2$	$-$	$6$	$z$	$y$	$+$	$=$	$c$
$\underbrace{\hspace{1.5cm}}$				$\underbrace{\hspace{1.5cm}}$					
$6$				$6$					

- $1$  – a **coefficient** - known number or value.
- $2$  – a **variable** - unknown number or value.
- $3$  – an **exponent** - power or inverse power (root).
- $4$  – an **operator** - arithmetical process.
- $5$  – a **constant** - standard value that can be found..
- $6$  – a **term** - grouped entity of 1, 2 and 3.

*Constants* are often used in a *formula*. Formulae are specific calculations used to solve common arithmetical problems, and they can be found in reference material.

## Rules of Algebra

Re-assembling equations as simpler arrangements relies on some basic rules that must be followed for consistent answers. The order in which operations are processed could alter the answer, as shown in the example below:

$3 + 4 \times 5$  could be  $3 + 4 = 7 \times 5 = 35$  but at the same time,  
 $3 + (4 \times 5)$  will be  $4 \times 5 = 20 + 3 = 23$

Note that adding brackets clears up any doubt as to which part of the calculation is meant as the first step to solve. Order rules of elementary algebra include:

*Order of operations:*

The general rule for order in calculating is by:

Step 1- calculations within **brackets (parenthesis)**.

Step 2- calculations of **exponents**.

Step 3- calculations of **multiplication**.

Step 4- calculations of **division**.

Step 5- calculations of **addition**.

Step 6- calculations of **subtraction**.

A simple aid to memory for order of calculations is **bemdas** or **pemdas**. Other rules apply with re-assembling equations including:

*Commutative rule:*

The *commutative rule* allows swapping around the notation positions on both sides of an equation to provide an easier calculation alternative, for instance with *addition*:

$$z + y = y + z \quad \text{or}$$

$$z + 3 = 3 + z$$

It also allows swapping the notation positions on each side of an equation to provide an easier calculation alternative, for instance with *multiplication*:

$$ab = ba \quad \text{or}$$

$$y3 = 3y$$

*Associative rule:*

The *associative rule* provides an alternative bracketed group to operate addition within an equation, for instance:

$$(a + b) + c = a + (b + c) \quad \text{or}$$

$$(a + 4) - 2c = a + (4 - 2c)$$

It also provides an alternative bracketed group to operate *multiplication* within an equation, for instance:

$$c(ba) = c(ba) \quad \text{or}$$

$$(a \times 2)3b = a(2 \times 3b)$$

*Distributive rule:*

The distributive rule allows multiplying to the sum of addition within a bracket (parenthesis) or to each quantity followed by adding, for instance:

$$\begin{aligned}a(b + c) &= ab + ac \text{ or} \\z(3 + 2y) &= 3z + 2zy\end{aligned}$$

*Operations with zero:*

When zero is added to any number it is not changed.

$$\begin{aligned}b + 0 &= b \\y + 0 &= y\end{aligned}$$

When zero is subtracted from any number it is not changed.

$$\begin{aligned}b - 0 &= b \\y - 0 &= y\end{aligned}$$

When any number is multiplied by zero the result is zero.

$$\begin{aligned}b \times 0 &= 0 \\y \times 0 &= 0\end{aligned}$$

If multiplying values equals zero then at least one of the values must be zero.

If  $ac = 0$  then  $a = 0$  or  $c = 0$

When zero is divided by any number it is the result is zero.

$$\frac{0}{y} = 0$$

A number cannot be divided by zero.

$$\frac{y}{0} = \text{the denominator cannot be 0.}$$

When a number is added to its opposite pair the result is zero.

$$\begin{aligned}b - b &= 0 \\y - y &= 0\end{aligned}$$

*Operations with negative quantities:*

Care must be taken when calculating with negative values and subtraction.

Subtraction can be considered as addition with a negative number, just as division can be thought of as inverse multiplication.

$$\begin{array}{llll}(-1) a & = & -a & (-1) 3 & = & -3 \\-(-a) & = & a & -(-3) & = & 3 \\(-a) b & = & -(ab) = a(-b) & (-3) y & = & -(3y) = 3(-y) \\(-a)(-b) & = & ab & (-3)(-y) & = & 3y \\-(a + b) & = & (-a) + (-b) & -(y + 3) & = & (-y) + (-3) = -y - 3\end{array}$$



### Fractions

The same rules as in fractions applies to algebraic calculations with fractions. (see [section 2.3](#)). These include:

When adding or subtracting with the same denominator, add or subtract the numerator variables.

$$\text{When } \frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$$

When adding or subtracting with the different denominators, find a common denominator and add or subtract the numerator variables.

$$\text{When } \frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$$

Balance (*equivalence*) is shown by *cross multiplying* unless b and d equal zero.

$$\frac{a}{b} = \frac{c}{d} \quad \text{Multiply all by } \frac{bd}{1} = \frac{abd}{b} = \frac{cbd}{d} = ad = cb$$

Negative signs can go anywhere in the fraction and two negatives equal a positive.

$$- \frac{a}{b} = \frac{a}{-b} = -\frac{a}{b} \quad \text{and} \quad \frac{-a}{-b} = \frac{a}{b}$$

If multiplying both top and bottom of a fraction by the same quantity maintains the fraction's same value, then the quantity must have been zero.

$$\text{If } \frac{a}{b} = \frac{ac}{bc} \quad \text{then } c = 0$$

When multiplying fractions multiply the numerators and multiply the denominators.

$$\text{When } \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

When dividing fractions, reverse the dividing fraction (the reciprocal) and multiply.

$$\text{When } \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

## Solving problems using algebra

The formula for the energy potential of mass is:

Example:

**$e = mc^2$**  where  $e$  is energy in joules  
 $m$  is mass in kilograms  
 $c$  constant, the speed of light (300,000,000 mtrs per sec)

**$e = 1 \text{ kilo} \times 300,000,000 \times 300,000,000 = 90,000$  million, million joules.**

Or  **$1 \text{ gram} = 90,000 \text{ million, million} \div 1000 = 90 \text{ million, million joules}$**

This is the incredible amount of energy available in 1 gram of matter available to unleash by nuclear weapons, equivalent to the energy of a 20 kiloton TNT bomb.

The formula for the volume of a cylindrical tank is given by  $v = \pi r^2 h$ .

Example:

**$v = \pi r^2 h$**  where  $v$  is volume of a tank  
 $\pi$  the constant  **$\pi$ ,  $22 \div 7 =$  approximately  $3.124$ .**  
 $r$  is the radius of a circle  
 $h$  is height of a tank

If we knew the radius of the tank was **0.6** mtrs and its height was **1.5** mtrs, then

**$v = \pi r^2 h = 3.142 \times 0.6 \times 0.6 \times 1.5 = 3.142 \times 0.36 \times 1.5 = 1.696$  cubic mtrs**

For practical purposes the tank's volume is rounded to approx. **1.7**

If the tank was only a third full (a sounding of **0.5** mtrs) then

**$v = \pi r^2 h = 3.142 \times 0.6 \times 0.6 \times 1.5 = 3.142 \times 0.36 \times 0.5 = 0.56 \text{ mtrs}^3$**

If we knew the volume of the tank (**2** mtrs<sup>3</sup>) and its radius (**0.4** mtrs) then its height is:

**$v = \pi r^2 h = \frac{v \times 1}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2} = \frac{v}{\pi r^2} = h = \frac{2}{3.142 \times 0.4^2} = 3.9783577$  mtrs**

The tank's height is rounded to approx. **3.98 mtrs.**

Note the process of rounding to 3 mtrs and 98 cms is 1.64 millimetres greater than our calculated answer, and for the purposes of refuelling will be within the required accuracy level. Note also that  **$\pi$**  is one of a few *irrational numbers*. It cannot be determined as a final number, for example:

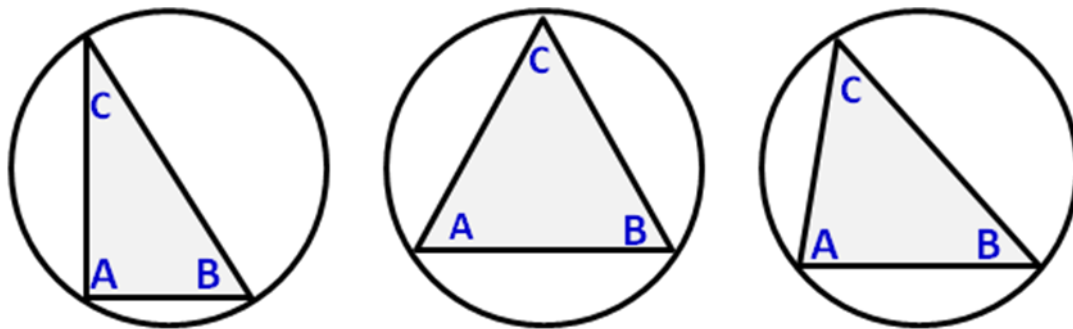
**$\pi = 22 \div 7 = 3.1428571428571428571428571????????$**  and so on.

## 2.5 Trigonometry

Trigonometry is a mathematical means of calculating the relationships between the length of the sides of a triangle and the angles between those sides.

### *Properties of triangles*

A circle can be drawn around any triangle to touch each of its three points. The sum of the internal angles of the points always adds up to  $180^\circ$ .

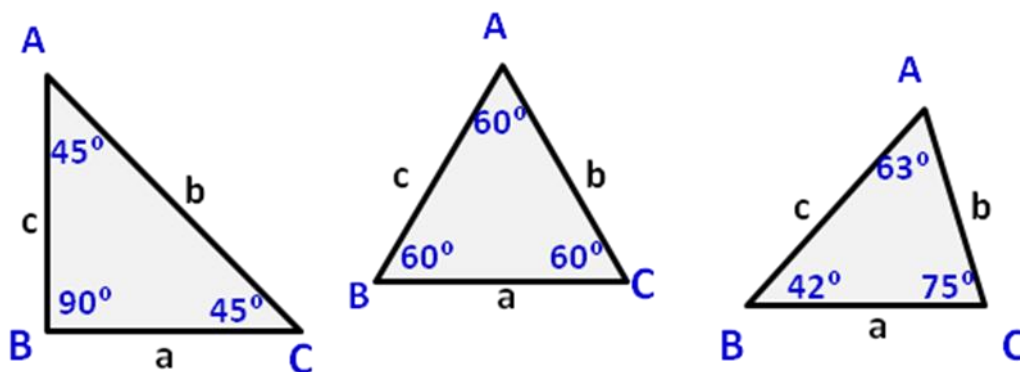


$$\text{Angles } A + B + C = 180^\circ$$

If we know any two angles then we can calculate the other. For example:

$$A \ 63^\circ + B \ 75^\circ = 138 \quad \text{therefore} \quad C = 180^\circ - 138 = 42^\circ$$

Triangles come in different shapes. They can have a  $90^\circ$  angle (right angle triangle), a  $90^\circ$  angle and two equal length sides (equilateral triangle), equal length sides and angles (isosceles triangle) or different length sides and angles (irregular triangle). An angle is designated by the symbol of  $\theta$ .



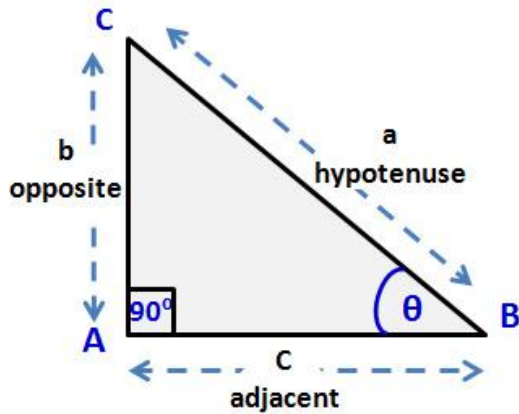
### **Opposite, adjacent and hypotenuse**

In this right angle triangle using  $\theta$  **ABC** as a reference point:

The opposite side is opposite  $\theta$ .

The adjacent side is adjacent (next to) to  $\theta$ .

The hypotenuse is the longest of the sides.



Trigonometry calculates an unknown side or angle of right angle triangles using the functions of Tangent, Sine and Cosine and formulas. In the past, the answers to all the sides' lengths and angles were measured and tabulated so solutions could be looked up in the books of tables. More often nowadays we use the function keys on an electronic calculator to do the same thing.

$$\text{Sine (sin) of } \theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{b}{a}$$

$$\text{Cosine (cos) of } \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{c}{a}$$

$$\text{Tangent (tan) of } \theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{b}{c}$$

These formulas can be remembered by the mnemonic of:

**SOH-CAH-TOA**

Alternately the reciprocal functions of SOH-CAH-TOA above can be used:

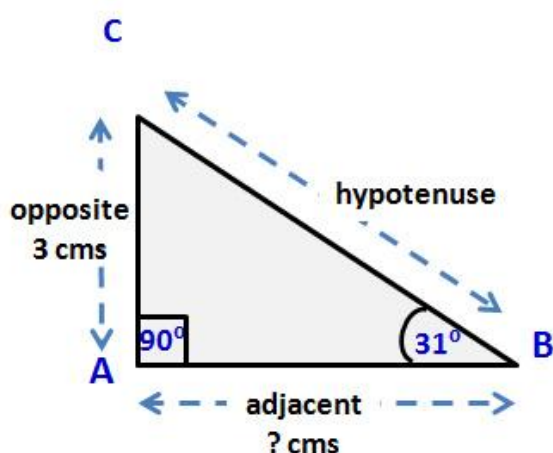
$$\frac{1}{\text{Sin}} \theta = \text{Secant (sec)} \theta = \frac{\text{Hypotenuse}}{\text{Adjacent}} = \frac{a}{c}$$

$$\frac{1}{\text{Cos}} \theta = \text{Cosecant (cosec)} \theta = \frac{\text{Hypotenuse}}{\text{Opposite}} = \frac{a}{b}$$

$$\frac{1}{\text{Tan}} \theta = \text{Cotangent (cot)} \theta = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{c}{b}$$

**Finding a length of a side in the triangle ABC below:**

If we know the length of a side and two angles of a right angle triangle we can calculate the other sides by the rules of trigonometry.



*Example 1:* (Finding the hypotenuse with the sine function of a scientific calculator)

<b>Sin</b>	<b>31</b>	<b>=</b>	<b>0.515</b>
------------	-----------	----------	--------------

Therefore,  $0.515 = \text{Opposite} \div \text{Hypotenuse}$  or  $\text{Hyp} = 3\text{cms} \div 0.515 = \mathbf{5.8\text{cms}}$

*Example 2:* (Finding the hypotenuse with the cosine function of a scientific calculator)

<b>Cos</b>	<b>31</b>	<b>=</b>	<b>0.857</b>
------------	-----------	----------	--------------

Therefore,  $0.857 = \text{Adjacent} \div \text{Hypotenuse}$  or  $\text{Hyp} = 5\text{cms} \div 0.857 = \mathbf{5.8\text{cms}}$

*Example 3:* (Finding the adjacent with the tangent function of a scientific calculator)

<b>Tan</b>	<b>31</b>	<b>=</b>	<b>0.6008</b>
------------	-----------	----------	---------------

Therefore,  $0.6008 = \text{Opposite} \div \text{Adjacent}$  or  $\text{Adjacent} = 3\text{cms} \div 0.6008 = \mathbf{5\text{cms}}$

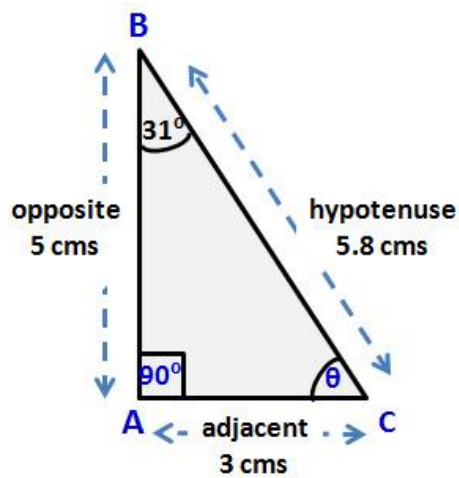
**Finding an angle in the triangle ACB below:**

If we know the length of two sides and two angles of a triangle we can calculate the other angle by the rules of triangles.

*Example:* We can find the last of the triangles angles (ACB) by the rules of triangles:

$$A \ 90^\circ + B \ 31^\circ = 121 \quad \text{therefore} \quad C = 180^\circ - 121 = 59^\circ$$

We could also find the last angle by the rules of trigonometry if we referenced our unknown angle  $\theta$ , (angle ACB) with the Opposite, Adjacent and Hypotenuse lengths.



*Example 1:* (Finding the  $\theta$  with the inverse tangent function of a scientific calculator)

$$\text{Opposite} \div \text{Adjacent} = \text{Tangent } \theta$$

5	÷	3	=	1.666	Shift	Tan	=	59°
---	---	---	---	-------	-------	-----	---	-----

*Example 2:* (Finding the  $\theta$  with the inverse sine function of a scientific calculator)

$$\text{Opposite} \div \text{Hypotenuse} = \text{Sine } \theta$$

5	÷	5.8	=	0.862	shift	Sin	=	59°
---	---	-----	---	-------	-------	-----	---	-----

*Example 3:* (Finding the  $\theta$  with the inverse cosine function of a scientific calculator)

$$\text{Adjacent} \div \text{Hypotenuse} = \text{Cos } \theta$$

3	÷	5.8	=	0.862	shift	Cos	=	59°
---	---	-----	---	-------	-------	-----	---	-----



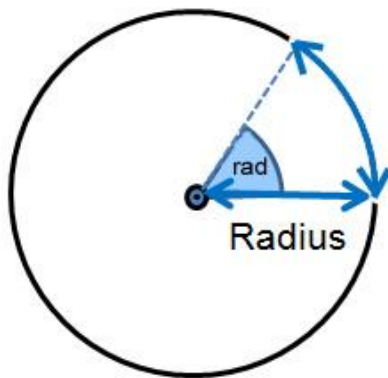
## ***Radians and properties of circles***

Up until now we have described angular measures by using degrees, there being  $360^\circ$  in a full circle. The dimensions in circles are found with the constant  $\pi$  (pi) that has an approximate value of  $22 \div 7 = 3.142$ . These dimensions include:

The area of a circle =  $\pi \times \text{radius}^2$ .

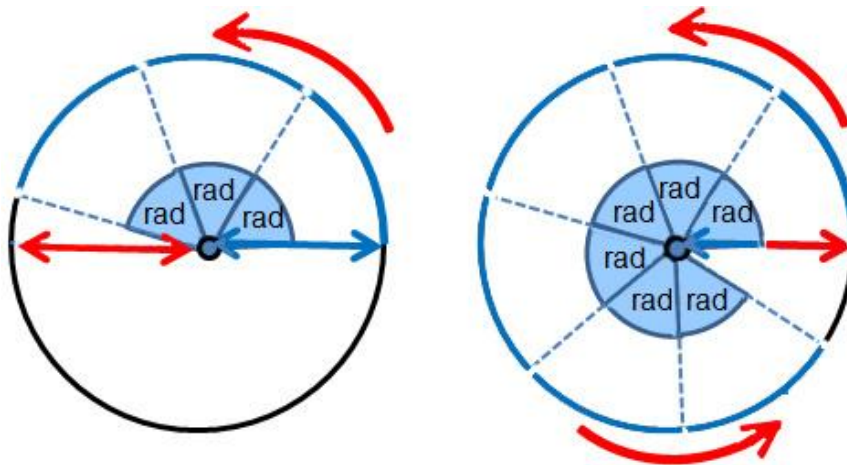
The circumference of a circle =  $2\pi \times \text{radius}$  ( $2\pi r$ ).

For solving rotational problems the radian as a measure is often preferred by mathematicians over the degree. A radian (rad) is the angle formed at the centre of any circle and an arc on its circumference that is the same length as the circle's radius. A radian is equal to  $57.295^\circ$ .



If the circle above is rolled through half a revolution its circumference will move through  $180^\circ$ . As the length of the circumference of a circle is  $2\pi \times \text{radius}$ , then it will also have moved through have this distance, or  $\pi$  radians = 3.142 radians.

If the circle above is rolled in one complete revolution its circumference will move through  $360^\circ$ . As the circumference is  $2\pi r$ , then it will also have moved through  $2\pi$  radians =  $2 \times 3.142 = 6.286$  radians.



\*Note -  $360^\circ \div 6.286 = 57.3^\circ$

Likewise,

$360^\circ = 2\pi$  radians,  $180^\circ = \pi$  radians,  $90^\circ = \pi/2$  radians

*Summary:*

1 radian per second = approximately 57.295 degrees per second

1 radian per second = approximately 9.5493 revolutions per minute (rpm)

0.1047 radian per second = approximately 1 rpm

One use of the *unit radian per second* is to calculate the power transmitted by a shaft (P) being the product of  $\omega$  (in radians per sec.) times the torque (T) in newton-meters applied to the shaft, or:

$$P = \omega \times T, \text{ in watts.}$$

As the radian is a dimensionless unit, the radian per sec. is dimensionally equivalent to the hertz, both being defined as  $\text{one s}^{-1}$ . Consequently care must be taken to avoid confusing angular frequency ( $\omega$ ) and frequency ( $\nu$ ).

## Chapter 3: Measuring

### 3.1 International System of Units

Measuring developed from comparison of commonly found weights and dimensions of local materials. During history the greatest empires spread their measurement systems furthest in support of their navigation, trade and colony building. Two main systems survive in the *Imperial System* and the *Metric System*; the qualities being Imperial with human scale (feet) and the Metric for easier calculations (decimal). Consequently, system conversions may be required in international trading and machine parts are not always interchangeable. To solve this, the metric based *International System of Units* (SI Units) has been adopted as a worldwide standard.

#### *The SI base measurements and symbols*

Length	<b>m</b> = metre	
Mass	<b>Kg</b> = kilogram	
Time	<b>s</b> = second	
Electric current	<b>A</b> = Ampere	
Luminous intensity	<b>cd</b> = candela	
Temperature	<b>K</b> = Kelvin	= The thermodynamic Kelvin scale uses the degree Celsius for its unit increments absolute zero or - 273·15° Celsius. Conversion $K = ^\circ C + 273\cdot 15$ $^\circ C = K - 273\cdot 15$
Amount of substance	<b>mol</b> = mole	In chemistry it expresses the amount of substance cwith as many atoms, molecules, ions, electrons as there are atoms in 12 grams of carbon-12 (relative atomic mass 12).

Measures are also found in multiples/derivations of SI measures, for example:

<b>Density</b> is measured as an object's	<b>Mass</b> per unit of <b>Volume</b>
<b>Electrical Power</b> is measured in	<b>Volts/Amps = Watts</b>
<b>Energy</b> is measured in	<b>Joules</b>
<b>Force</b> is measured in	<b>Newtons</b>
<b>Pressure</b> is measured in	<b>Pascals</b>
<b>Stress</b> is measured in	<b>Newtons/ metre<sup>2</sup></b>
<b>Torque</b> is measured in	<b>Newton/metres</b>
<b>Speed</b> is measured in	<b>Metres/second</b>
<b>Velocity</b> is measured in	<b>Metres/second</b> in a specified direction.

<b><i>Multiples of Units</i></b>								
Length	millimetre	mm	x 1000 =	metre	m	x 1000 =	kilometre	k
Mass	gram	gm	x 1000 =	kilogram	kg	x 1000 =	tonne	T
Force	Newton	N						
Pressure	Pascal	Pa	x 1000 =	hecta pascal	hPa	x 1000 =	mega pascal	mPa
Current	Ampere	A	x 1000 =	kilo amp	kA	x 1000 =	mega amp	mA
Power	Watts	W	x 1000 =	kilo watt	kW	x 1000 =	mega watt	mW
Energy	joule	j	x 1000 =	kilo joule	kj	x 1000 =	mega joule	mj
Luminous	candela	cd	x 1000 =	kilo candela	kcd	x 1000 =	megacandela	cd
Time	second	s	x 60 =	minute	m	x 60 =	hour	h
Nautical distance	metres	m	x 1852 =	mile	nm	nautical miles per hour are termed knots		kt

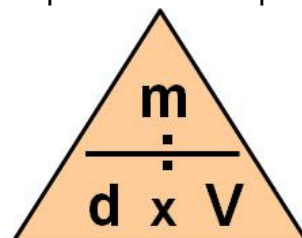
### 3.2 Derived units and formulas

Standard formulas simplify working with derived units and measures. However, some non SI unit measures commonly persist, for instance, nautical miles. When calculating, pay attention to whether the units are SI units and whether they are consistent for all *variables* or *constants* used within a formula.

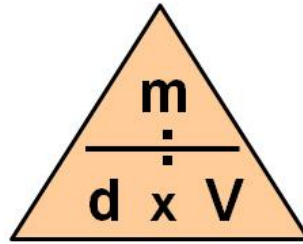
**Density is measured as an object's Mass per unit of Volume.**

The SI units for density are kg/m<sup>3</sup>. The imperial units are lb/ft<sup>3</sup> (slugs/ft<sup>3</sup>). Though pounds per cubic foot are used as an equivalent measure of density, pounds are actually a measure of force, not mass. Slugs are the correct measure of mass. Multiply slugs x 32.2 for an approximate value in imperial pounds.

\*Note - although equivalent for sea level calculations, technically *weight* unlike *mass* is an object's measured downward pressure at a specified gravitational locality.



The formulas of mass, density and volume are expressed within the triangle as:



The formulas of mass, density and volume are expressed within the triangle as:

$$d = \frac{m}{V} \qquad m = d \times V \qquad V = \frac{m}{d}$$

Where:

$d$  = density (meters<sup>3</sup>/kilogram, feet<sup>3</sup>/slug)

$m$  = mass (kilograms, slugs)

$V$  = volume (metres<sup>3</sup>, feet<sup>3</sup>)

*Specific Gravity* or *Relative Density* of a substance is a comparison of the mass of a volume of one substance to the mass of an equal volume of pure water. *Specific Gravity* is the mass compared to pure water valued as 1 (it is a ratio so it has no units). *Relative Density* is expressed as a percentage of pure water's mass.

$$\text{S.G.} = \frac{\text{mass of substance}}{\text{mass of fresh water}} = \text{R.D.\%}$$

*Example 1:*

What is the mass of a cubic meter of fresh water? What is the mass of a cubic meter of salt water? How much salt is in a cubic meter of salt water?

*Fresh water* SG 1.000-  $1\text{m}^3 = 1000 \text{ litres} \times \text{SG } 1.000 = 1000 \text{ kilograms} = 1 \text{ tonne}$

*Salt water* SG 1.025-  $1\text{m}^3 = 1000 \text{ litres} \times \text{SG } 1.025 = 1025 \text{ kilograms} = 1.025 \text{ tonne}$

*Salt water* **1025 kilograms** - *Fresh water* **1000 kilograms** = **25 kilograms**

*Example 2:*

What is the mass of fuel in a tank that measures 4 metres in length, 2 metres wide and 1.8 metres deep if the fuel has a *specific gravity* 80% that of fresh water?

$$\begin{aligned} \text{Volume} &= \text{Length} \times \text{Width} \times \text{Depth} \\ &= 4 \text{ metres} \times 2 \text{ metres} \times 1.8 \text{ metres} \\ &= 6.4\text{m}^3 \text{ (cubic metres)} \end{aligned}$$

$$\begin{aligned} \text{Mass (tonnes)} &= \text{Volume (cubic metres)} \times \text{Specific Gravity} \\ &= 6.4\text{m}^3 \times 0.8 \\ &= 5.12 \text{ tonnes} \end{aligned}$$

**Example 3:**

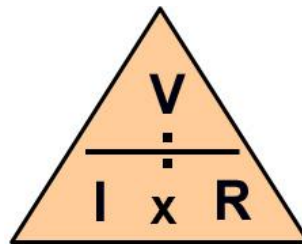
If oil weighs 860 kgs/m<sup>3</sup>, what is its *specific gravity*?

\* Though the terms are used

$$\begin{aligned}
 \text{S.G. (R.D)} &= \frac{\text{mass of substance}}{\text{mass of fresh water}} \\
 &= \frac{860}{1000} \\
 &= 0.86
 \end{aligned}$$

**Electrical Power-** Electricity is measured in units described by *Ohm's law*:

the *current* through a conductor between two points is directly proportional to the *potential difference* across the points, and inversely proportional to the *resistance* between them”.



$$I = \frac{V}{R}$$

$$V = I \times R$$

$$R = \frac{V}{I}$$

Where:

*I* = current (ampere, A)

*V* = potential difference (volt, V)

*R* = resistance (ohm, Ω)

**Example 1:**

If a 6 Ω resistor is placed in a 12V circuit, what will the current be?

$$I = \frac{V}{R} = \frac{12}{6} = 2 \text{ Amps}$$

**Example 2:**

If a 3 amp motor has 4 Ω resistance, what will the voltage be?

$$V = I \times R = 3 \times 4 = 12 \text{ Volts}$$

*Example 3:*

In a 12V circuit what resistance will a 1 Amp radio develop?

$$R = \frac{V}{I} = \frac{12}{1} = 12 \text{ Ohms}$$

Electrical **power** is termed **watts** and is calculated as  $P = V \times I$

Where:

$P$  = power (watts)

$I$  = current (ampere, A)

$V$  = potential difference (volt, V)

*Example 4:*

In a 12V circuit, what power will a 2 amp globe use?

$$P = V \times I = 12 \times 2 = 24 \text{ Watts}$$

**Energy is measured in Joules**

The joule (j) is defined as the energy of work or heat equal expended in applying a force of one newton through a distance of one metre (Nm), or in passing an electric current of one amp through a resistance of one ohm for one second.

$$j = \frac{\text{Kg} \times \text{m}^2}{\text{s}^2} = \text{N} \times \text{m} = \text{Pa} \times \text{m}^3 = \text{W} \times \text{s} = \text{C} \times \text{V}$$

Where:

J is a joule,

kg is kilogram,

m is metre,

s is second,

N is newton,

Pa is pascal,

W is watt,

C is coulomb,

V is volt.

In electrical terms, the joule can also be defined as the work required to move one coulomb of electric charge through a potential difference of one volt (a *coulomb volt*). Similarly a volt is defined as the work required to produce one watt of power for one second (a *watt second*).

A microjoule equals one millionth ( $10^{-6}$ ) of a joule.

A kilojoule equals one thousand ( $10^3$ ) joules.

A megajoule equals one million ( $10^6$ ) joules.

*Converting joules into watts:*

You can calculate watts from joules and seconds, but you can't convert joules to watts, since joule and watt units represent different quantities.

$$\text{Power (W)} = \text{Energy (J)} \div \text{time (s)} \quad \text{or} \quad \text{Watt} = \text{joule} \div \text{second}$$

Where:

$P$  is power in watts (W),  $E$  energy in joules (J), time is *seconds* (s)

*Example:*

What is the power consumption of an electrical circuit that has energy consumption of 120 joules for time duration of 10 seconds?

$$\text{Watt} = \text{joule} \div \text{second}$$

$$\text{Watts} = 120 \div 10 = 12 \text{ W}$$

*Converting watts into joules:*

$$\text{Energy (j)} = \text{Power (W)} \times \text{time (s)} \quad \text{or} \quad \text{joules} = \text{Watts} \times \text{seconds}$$

*Example:*

What is the energy consumption of an electrical circuit that has power consumption of 12 watts for time duration of 10 seconds?

$$\text{joules} = \text{Watts} \times \text{seconds}$$

$$\text{joules} = 12 \times 10 = 120 \text{ j}$$

**Force is measured in *Newtons* and describes motive energy.**

One Newton of force is required to accelerate one kilogram by one metre per second per second. A force over a given area is described as **stress**.

**Pressure is measured in *Pascals***

**Stress is measured in *Newtons/ metre<sup>2</sup>***

**Torque is measured in *Newton/Metres* and describes angular/rotating motion.**

The power of a rotating body is expressed as:

$$P = T w$$

where

$P$  = power

$T$  = torque or moment (N/m)

$w$  = angular velocity (rad/s)

$$1 \text{ rad} = 360^\circ / 2\pi = 57.295^\circ$$



Torque of a body in angular motion is expressed as:

$$T = I a$$

where

$I$  = moment of inertia ( $\text{kg m}^2$ )

$a$  = angular acceleration ( $\text{rad/s}^2$ )

The Newton metre as a unit of torque (a *moment*) is equal to one Newton applied perpendicular to a moment arm of one metre long. But it is also a unit of energy/work equivalent to the joule. For energy/work usage, the metre is the distance moved in the direction of the force rather than the perpendicular distance from a fulcrum of torque. Since torque is energy expended per angle of revolution, a Newton metre of torque is equivalent to a joule per radian. Seen below, Newton metres and joules are dimensionally equivalent as can be calculated by the same expression:

$$1 \text{ Nm} = \frac{\text{Kg} \times \text{m}^2}{\text{s}^2} \qquad 1 \text{ j} = \frac{\text{Kg} \times \text{m}^2}{\text{s}^2}$$

However, Nm and j must be distinguished in order to avoid misunderstanding torque and energy the same. Similar dimensionally equivalent units are Pa versus  $\text{j/m}^3$ .

### Conversion factors

1 newton metre = 0.7375621 pound-feet (often "foot-pounds")

1 kilogram-force metre = 9.80665 Nm

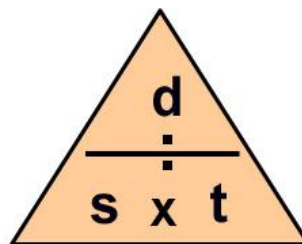
1 pound-foot (foot-pound)  $\equiv$  1 pound-force-foot = 1.3558 N·m

1 inch-ounce-force = 7.0615518 mNm

1 dyne-centimetre =  $10^{-7}$  Nm

### Speed is the rate at which distance is travelled.

The formulas of speed, distance and time are expressed within the triangle as:



$$s = \frac{d}{t}$$

$$d = s \times t$$

$$t = \frac{d}{s}$$

Where:

$s$  = speed (metres/second, kilometres/hour, knots)

$d$  = distance (metres, kilometres, nautical miles)

$t$  = time (seconds, hours)

When calculating use compatible units, for example:

$m/s = \text{metres} \div \text{seconds}$

$k/hr = \text{kilometres} \div \text{hours}$

$\text{knots} = \text{nautical miles} \div \text{hours}$

*Example 1:*

A vessel travels 15 nautical miles in three hours. What is its speed?

$$s = \frac{d}{t} = \frac{15}{3} = 5 \text{ Knots}$$

*Example 2:*

How far does a vessel travel if she steams at 10 knots for 4 hours?

$$d = s \times t = 10 \times 4 = 40 \text{ nautical miles}$$

*Example 3:*

How long will a vessel take to travel 60 nm at 5 knots?

$$t = \frac{d}{s} = \frac{60}{5} = 12 \text{ hours}$$

**Velocity** is speed in a specific direction. **Acceleration** is a measure of the change in speed of a body expressed as:

$$\text{acceleration } m/s^2 = dv \div dt = (v_2 - v_1) \div (t_2 - t_1)$$

Where:

dv = change in velocity

$v_2$  = final speed (m/s, ft/s)

$v_1$  = initial speed (m/s, ft/s)

dt = time taken (s)

$t_2$  = final time (s)

$t_1$  = initial time (s)

**Temperature is measured in scales Kelvin, Celsius (centigrade) or Fahrenheit.**

The thermodynamic Kelvin scale uses the degree Celsius for its unit increments absolute zero or  $-273.15^\circ$  Celsius. Conversion:

$$K = ^\circ C + 273.15 \quad ^\circ C = K - 273.15$$

### 3.3 Measuring Tools

*Some illustrations in this section contain enhanced drawings courtesy of ANTA Publications*

#### Measuring accurately

Measuring tools will be damaged if dropped. Tools that give readings of less than 0.1 mm can be put out of adjustment by poor handling. They must be *calibrated* regularly by checks against a standard of measurement (a precisely made gauge) to ensure accuracy. Steel expands with temperature rise so gauge accuracy is also affected by such change. If using screw pitch, radius or form gauges you should sight against a light background to see differences between the work piece and the gauge clearly.

#### Care and use

When using all measuring instruments the following precautions should be observed:

- Never drop the instrument
- When not in use replace in its case or clean rag, never on a hard steel bench
- Never allow dirt, filings, cutting oils or any other foreign substances to come in contact with the instrument
- Do not put the instrument on top of or under other tools
- Never measure moving objects
- Zero the instrument before use

#### Storage Procedures

To ensure long and reliable service:

- Clean the instrument thoroughly during and after use
- Lightly oil or wrap the instrument in oiled paper
- Store the instrument in its own case protected from outside damage
- Store the instrument in a dry place away from corrosive chemicals or solvents

## Standard Gauges

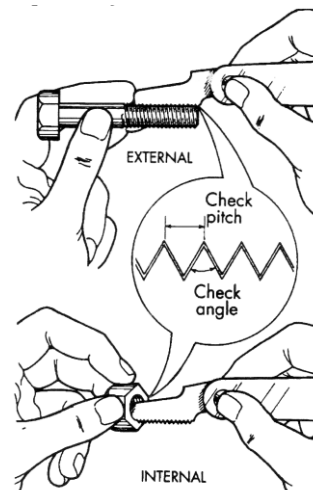
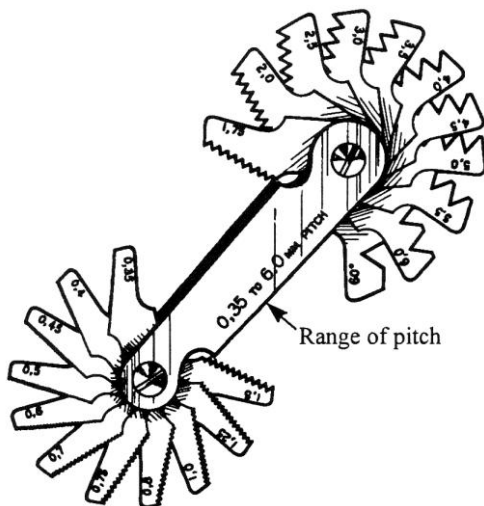
### Screw Pitch Gauge

A screw pitch gauge is a collection of blades of differing teeth sizes enabling both the pitch and form (*shape*) of threads to be checked. Each blade has a thread form stamped on it being in Metric, Whitworth, BSF, UNF or UNC.

Before measuring, assess the approximate pitch with a ruler. For metric threads:

- Line up a major division on the ruler with a crest of the thread.
- Count the number of crests to another major division, (distance 20 - 30 mm).
- Divide the chosen length by the number of crests counted.

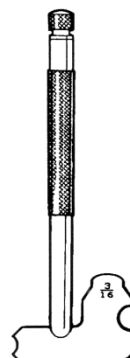
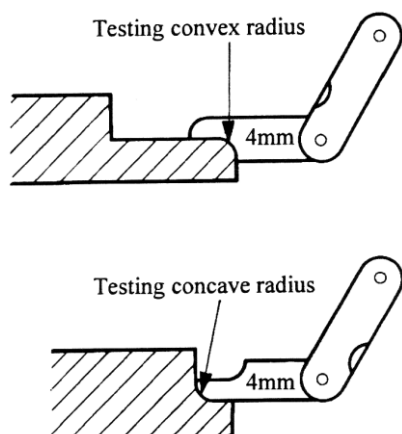
That result is roughly the thread pitch. Next choose the gauge blade closest to this for a fine check.



Use the same method over a distance of one inch for imperial threads as they are measured in threads per inch (TPI).

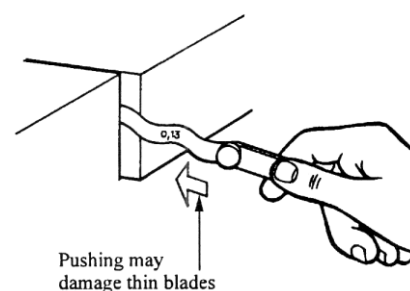
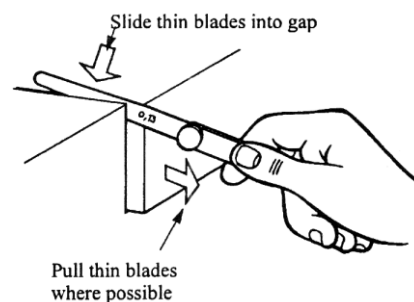
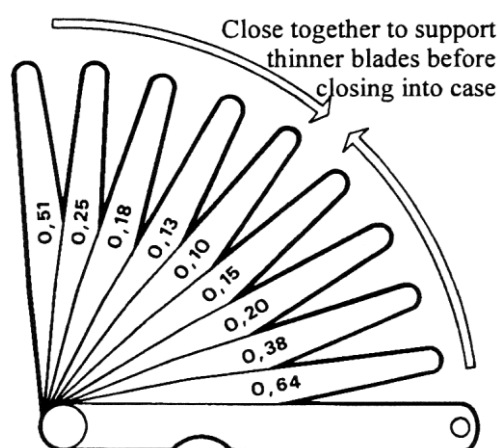
## Radius Gauge

This gauge is a set of blades shaped with matching convex and concave radii. They are used to check internal and external radii. Radius gauges of less than  $90^\circ$  may be called fillet gauges.



## Feeler Gauge

This gauge is a set of differing thickness blades (typically 0.05 mm-1mm) used to measure the width of small grooves or set clearances between mating parts. The blades can be doubled up to measure a greater thickness if required. Care should be taken with thinner blades to pull through any gap rather than pushing, in order to avoid blade the kinking with consequent damage.



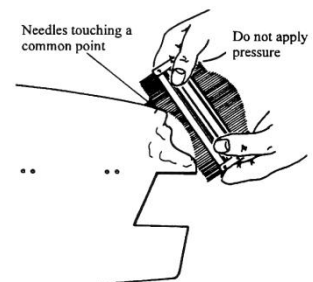
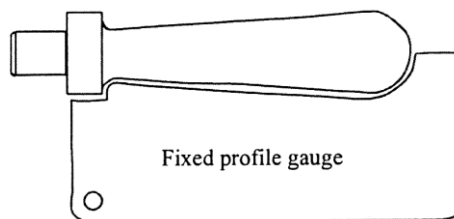
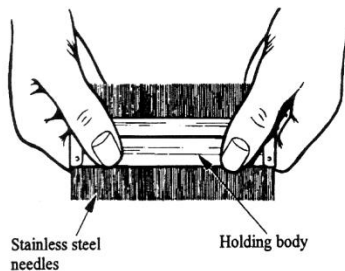
### ***Thickness Gauge***

This gauge is used to measure the thickness of sheet materials such as paper, plastics, cardboard or leather and sheet metals. Exposure to dirt and moisture should be avoided by returning to their storage box immediately after use.



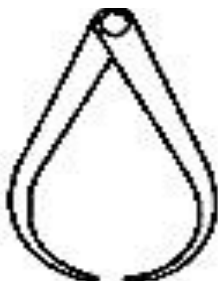
### ***Form or Profile Gauge***

These can be fixed or adjustable types, as below, used to check work pieces shape. The adjustable type is set to a master shape to compare with the work piece.



### ***Callipers***

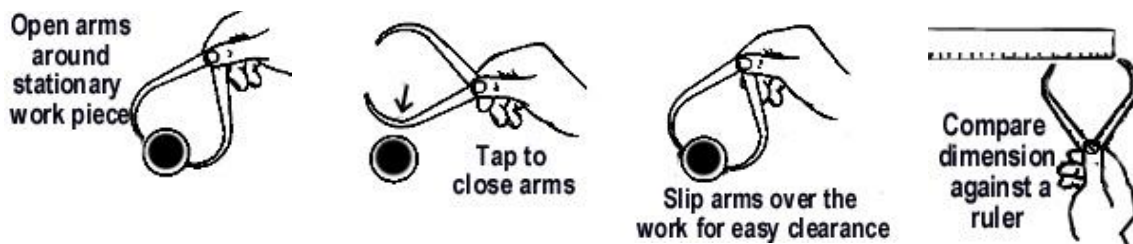
Various models all transfer a measure from a standard piece to a work piece. Two arms are hinged enabling a chosen gap to be set. Some are spring-loaded joint with an adjusting screw. Accurate measurement when using callipers depends upon the tightness of the calliper's arms against the work piece. Skill is needed to obtain the correct light pressured feel on the callipers as they slip over the work.



### ***Using outside callipers***

Outside callipers are used for outside diameters/dimensions or to check whether external surfaces are parallel. To use:

- Open the arms until they pass around the diameter to be measured. The work piece must be stationary.
- Gently tap the back of one arm against a solid part of the work piece to slightly close up the arms.
- Try the new setting with the callipers at right angles to the work. Continue to check and adjust the callipers until you feel the arms just slip over the work.
- Lastly compare the arms' gap with a steel rule on a flat smooth surface



### ***Using inside callipers***

Inside callipers are used for inside diameters/dimensions or to check whether internal surfaces are parallel. To use:

- Place an arm of the callipers just inside the bottom of the hole to be measured.
- With the adjusting screw, open the other arm so it touches the top of the hole.
- Rock the callipers slightly on the lower arm and adjust the screw until you feel the callipers tight within the hole.
- Wriggle the callipers within the hole to ensure that the arms are a tight fit all around.

## Graduated Measuring Tools

### Steel Rules

Steel rules can measure accurately to  $\pm 0.5$  mm if maintained and read properly.

6 inch



12 inch



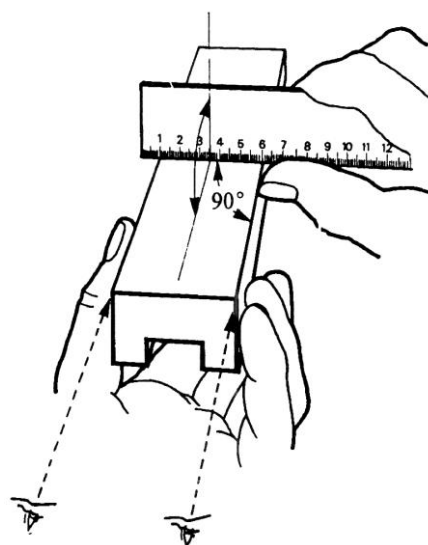
150 millimetre



300 millimetre



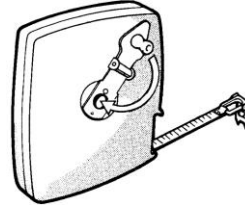
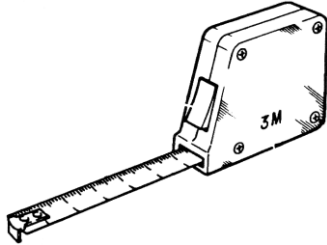
The ruler must be maintained with square and sharp edges. A common error called parallax is caused by sighting at an angle to the scale, rather than at right angles.





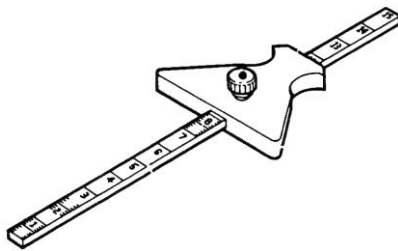
## Measuring Tapes

The tape is subject to damage so it must be cleaned as it self-retracts into the housing. Salt water or corrosive/sticky contaminants can ruin a tape and obliterated the scale markings rendering reading difficult.



## Depth Gauge

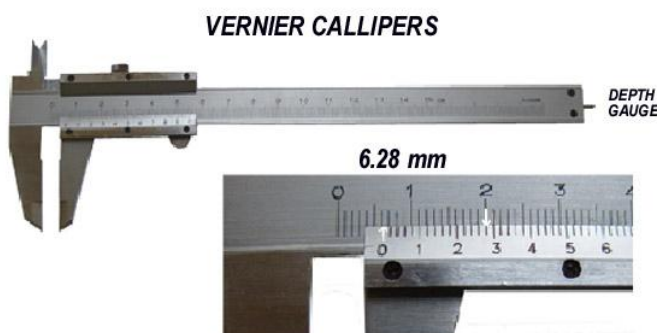
This gauge is used to measure the depth of holes or the distance between surfaces.



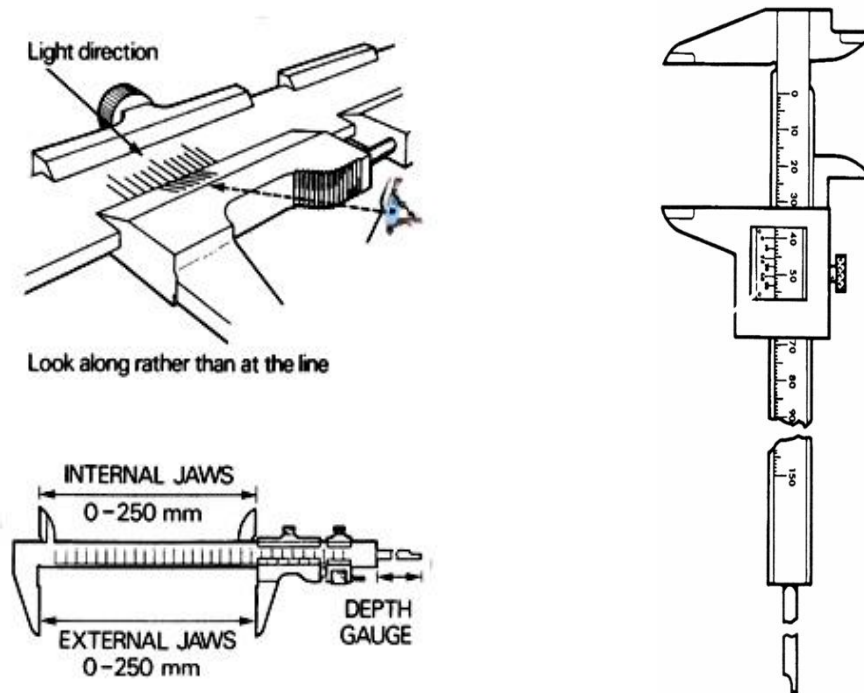
## Vernier Callipers & Micrometers

### Verniers

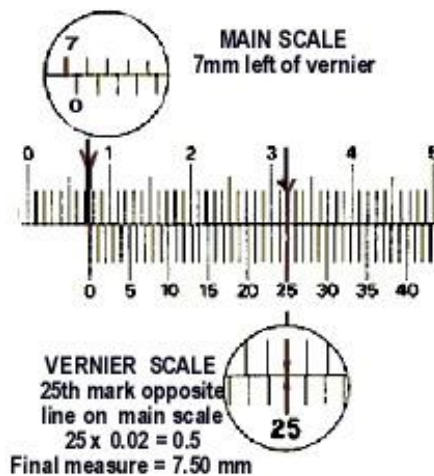
Used for internal, external or depth measurement, they have scales of up to 250 mm. Standard callipers measure to within 0.02 mm (0.001 in). Digital callipers are have accuracy to 0.01 mm (0.0005 in). They must be stored in a clean, dry place (preferably in their protective case). The jaws must be protected against damage.

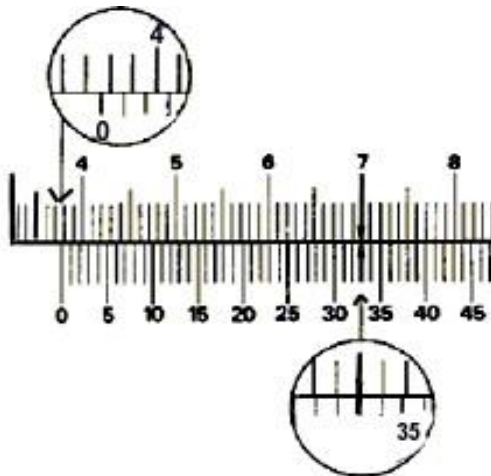


*Reading a Vernier* – They have a *main scale* providing a course measure and a *vernier scale* for the remainder fine measure. An experienced user can roughly estimate the fractional remainder, and consequently where to expect the vernier scale markings to align. Sighting across rather than over the marks will facilitate taking a reading. Move to a position where the light comes from behind the vernier scale at the same angle as your line of sight.



Read the main scale to the left from the metric vernier zero to find the whole number (as below 7 mm). To read the remaining fraction, look at the vernier scale. Note which of the vernier marks align with a main scale mark. Each of the vernier scale marks represents 0.02 mm. Finally multiply the number of marks on the vernier scale by 0.02 and add the result to the reading of the main scale, in this case 7.50 mm.

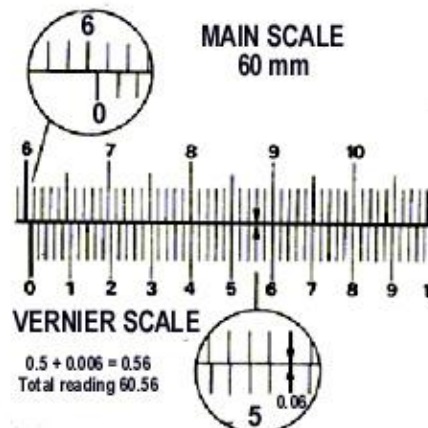
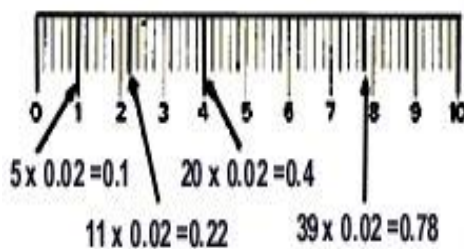




The drawing above shows a metric vernier reading just over 37 marks on the main scale to the left of the vernier scale's zero, or 37+ millimetres. The 33rd mark on the vernier scale aligns with a main scale mark giving  **$33 \times 0.02 = 0.66 \text{ mm}$**

Now add  **$0.66 \text{ mm}$**  to the main scale reading of  **$37 \text{ mm}$**  to give a total of  **$37.66 \text{ mm}$** .

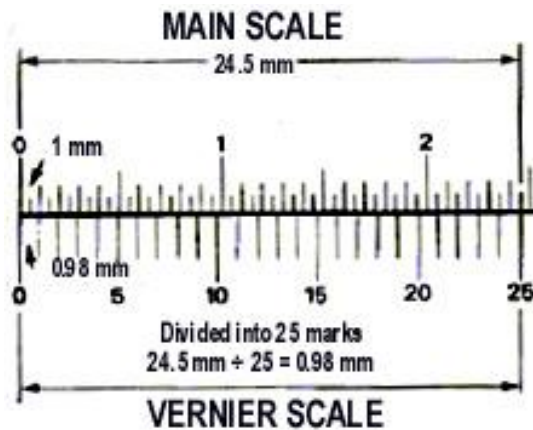
Some metric verniers with a 49 mm long scale have each fifth mark of the vernier scale numbered from 1 to 10. As each mark on the vernier scale represents 0.02 mm, then the fifth mark of ( $5 \times 0.02 = 0.1 \text{ mm}$ ) is numbered 1. The tenth mark is marked 2, the fifteenth mark marked 3, and so on.



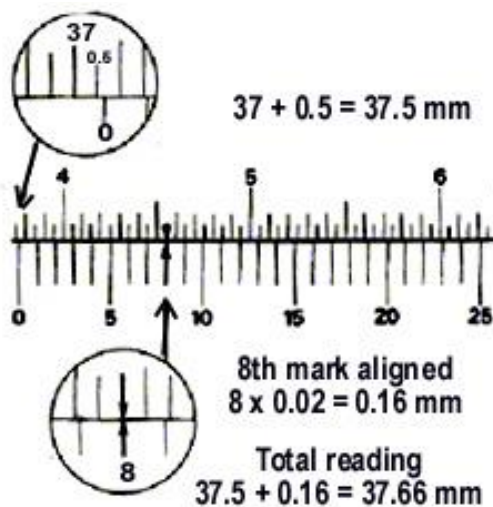
With this type of scale read the main scale as before, read the numbered divisions of the vernier scale as tenths of a millimetre then complete the reading by adding the extra 0.02 lines. In the main scale above reads 60 millimetres. The vernier shows the fifth mark which represents 0.5 mm, plus 3 extra divisions which represent:

**$3 \times 0.02 = 0.06 \text{ mm}$**  Total reading is  **$60 + 0.5 + 0.06 = 60.56 \text{ mm}$**

Some metric verniers have their main scale divided into millimetres and half millimetres. The vernier scale is divided into 25 equal marks (24.5 mm long). The length of each vernier division is therefore one twenty-fifth of 24.5 mm (0.98 mm), as shown below.



*Example* -The tool below has a vernier scale 24.5 mm long with 0.02 mm vernier scale readings (the 25 marks represent from 0mm - 0.5 mm).



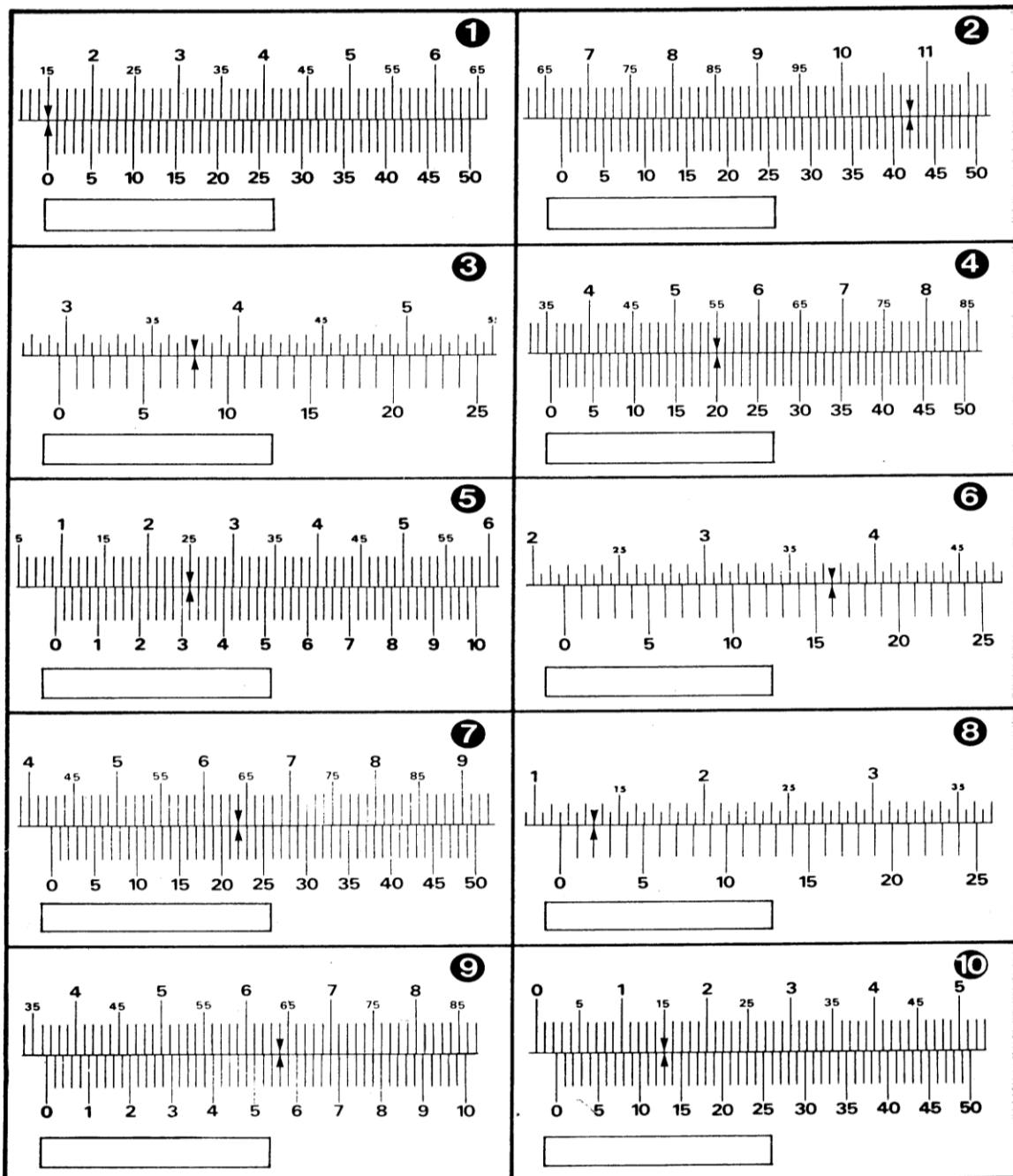
There are 37 major divisions on the main scale to the left of the zero, which equals 37 mm. There is also one half-millimetre division which equals 0.5 mm.

$$37 + 0.5 = 37.5 \text{ mm}$$

The eighth mark on the vernier's scale aligns with a mark on the main scale. Multiply 8 by 0.02 which represents 0.16 and add this to the main scale reading.

$$\text{Total reading} \quad 37.50 + 0.16 = 37.66 \text{ mm}$$

Check your progress reading a vernier - try these:



Drawing courtesy of Anta publications

Answers reading a vernier - as below:

- |  |   |
|--|---|
| 1. 15mm  | 2. $66\text{mm} + (42 \times 0.02=0.84) = 66.84\text{mm}$   |
| 3. $29.5\text{mm} + (8 \times 0.02=0.16) = 29.66\text{mm}$ | 4. $35\text{mm} + (20 \times 0.02=0.4) = 35.4\text{mm}$     |
| 5. $9\text{mm} + (16 \times 0.02=0.32) = 9.32\text{mm}$    | 6. $21.5\text{mm} + (16 \times 0.02=0.32) = 21.82\text{mm}$ |
| 7. $42\text{mm} + (22 \times 0.02=0.44) = 42.44\text{mm}$  | 8. $13.5\text{mm} + (2 \times 0.02=0.04) = 13.54\text{mm}$  |
| 9. $36\text{mm} + (28 \times 0.02=0.56) = 36.56\text{mm}$  | 10. $2 + (13 \times 0.02=0.26) = 2.26\text{mm}$             |

## Micrometers

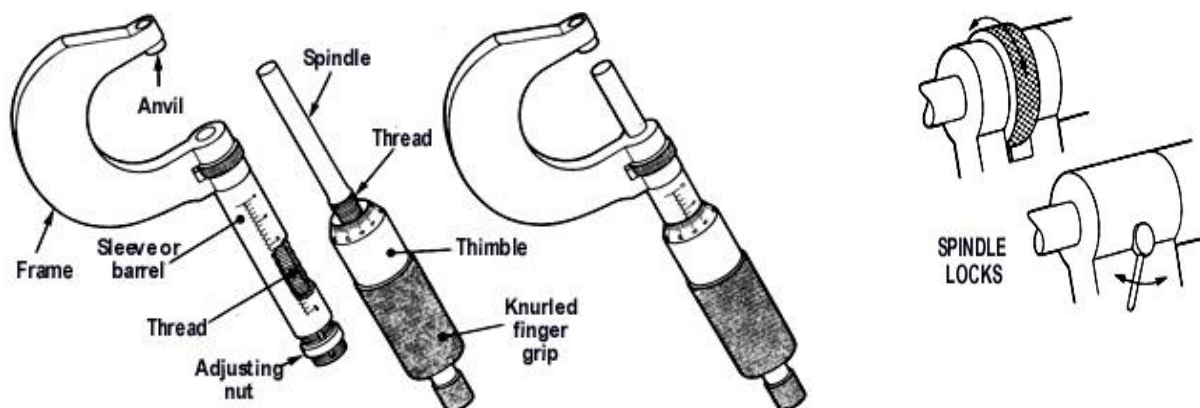
Outside micrometers enable very accurate measurement of diameters, thickness and length. All having a measuring range limited by the size of frame and length of the spindle's thread, typically 0 to 25 millimetres. Larger opening frames (50 mm) can use a removable 25 mm spacer insert to double the range while still using a standard 25 mm thread length.



In the drawing below, the main parts of a micrometer are:

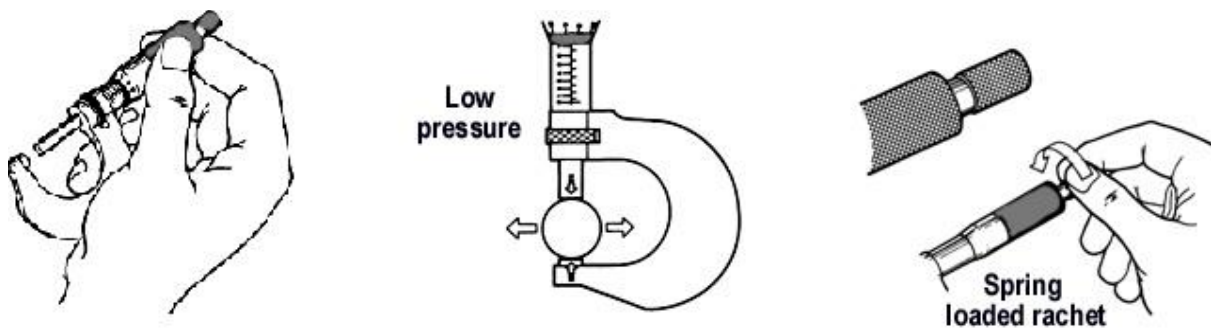
Frame – Anvil – Spindle & Thread – Sleeve or Barrel – Thimble.

In use, the anvil is held against the work piece while the spindle is wound inwards to make a tight fit. A knurled collar or lever locks the spindle in the barrel. The dimension is read from the scale.



Skill is needed to get accurate measures with a micrometer. Excessive pressure taking readings will give lower readings, strain the thread and distort the frame. The anvil is set against a work piece and the finger grip turned to tighten the spindle. A fine adjuster may be fitted, to limit over tightening. Turn it until the spring-loaded torque ratchet audibly clicks. Finally, tighten the spindle lock to stop movement when reading the scale. Remember to loosen it before using again.

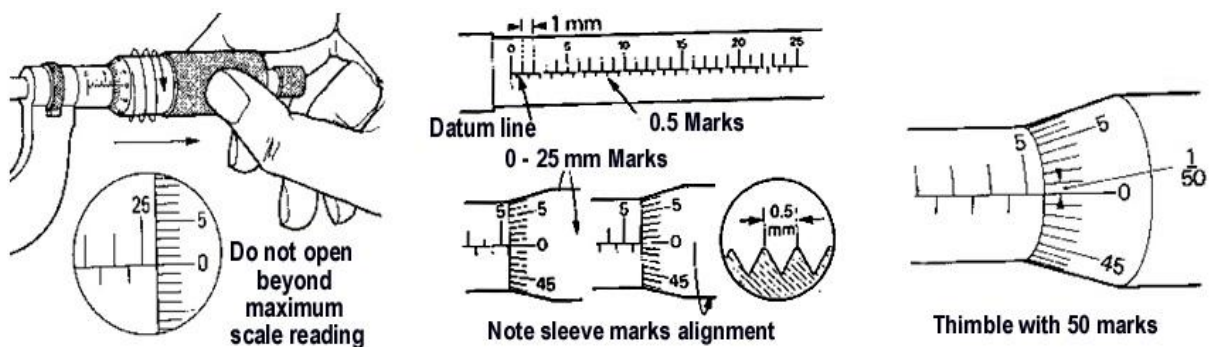




### **Checking a micrometer with scale to 0.01 of a millimetre**

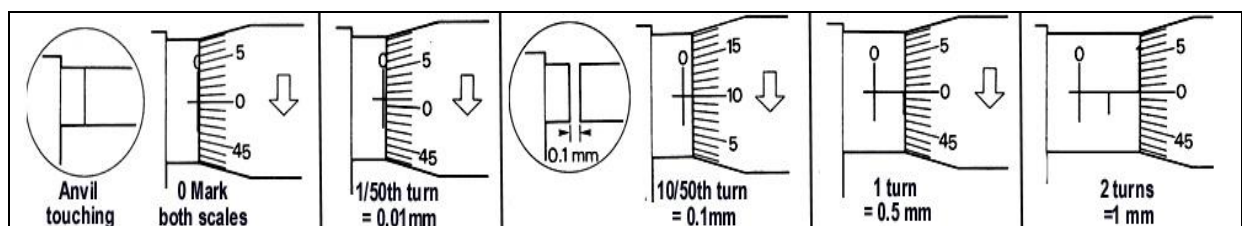
Keeping the marks on the sleeve towards you, hold the micrometer's frame with one hand. Use the other hand to screw the knurled part of the thimble anti-clockwise. This moves the spindle to uncover the marks on the sleeve.

The gap between the anvils should equal to the uncovered length of the datum line. The datum line on the sleeve is marked in millimetres and half millimetres, from zero to 25 mm, and usually each fifth millimetre is numbered. Turn the thimble until zero is level with the datum line. Note the position of the mark on the sleeve.



Turn the thimble one complete turn. The thimble will move along one graduation of the sleeve scale. This is because the pitch of the thread on the spindle is half a millimetre. There are 50 marks and each fifth mark is numbered. Two turns of the thimble move the spindle one millimetre.

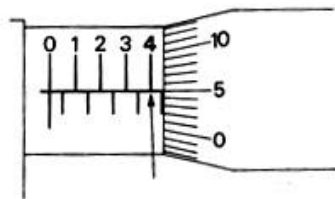
Now wipe the face of the anvils with a piece of clean cloth. Screw the thimble inwards towards the frame until the anvils are touching. Both scales should both read zero. If they are not then you need to get the tool repaired. If the error is slight and consistent then you can mathematically correct all measures from that tool.



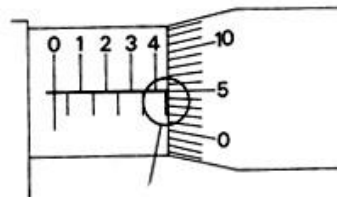
### **Reading a metric micrometer**

*Example 1* - Read the number of fully visible millimetre and half millimetre marks on the barrel. Read the thimble scale number that aligns with the datum line. Total the readings, as below:

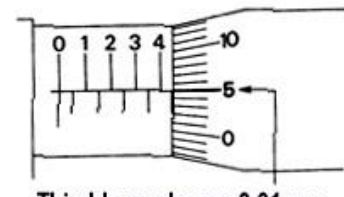
$$4.0 \text{ mm} + 0.5 \text{ mm} + 0.05 \text{ mm} = 4.55 \text{ mm}$$



Major barrel marks are 1 mm



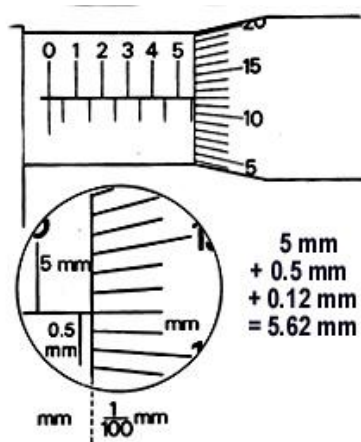
Minor barrel marks are 0.5 mm



Thimble marks are 0.01 mm  
 $4 + 0.5 + 0.05 = 4.55 \text{ mm}$

*Example 2* - Read the number of fully visible millimetre and half millimetre marks on the barrel. Read the thimble scale number that aligns with the datum line. Total the readings, as below:

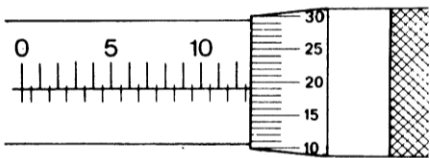
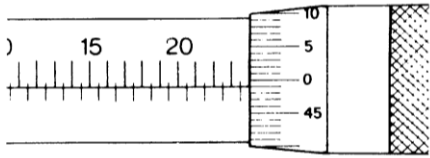
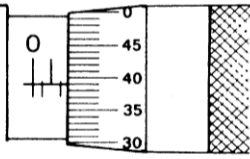
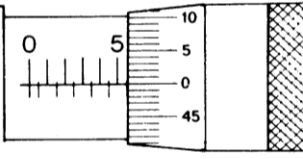
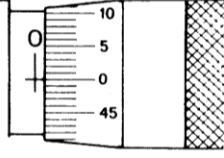
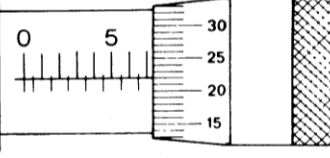
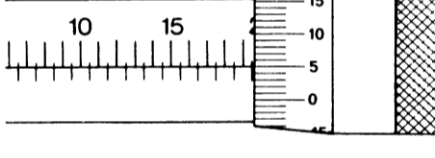
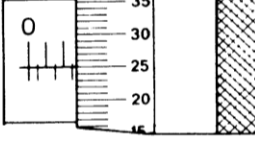
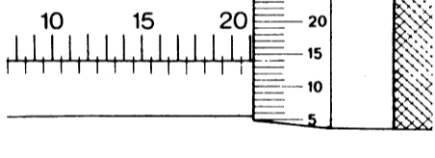
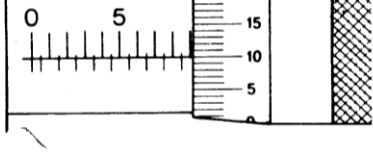
$$5.0 \text{ mm} + 0.5 \text{ mm} + 0.12 \text{ mm} = 5.62 \text{ mm}$$



$$\begin{aligned} &5 \text{ mm} \\ &+ 0.5 \text{ mm} \\ &+ 0.12 \text{ mm} \\ &= 5.62 \text{ mm} \end{aligned}$$



Check your progress reading a micrometer - try these:

<p><b>1</b></p>  <p><input type="text"/></p>	<p><b>2</b></p>  <p><input type="text"/></p>
<p><b>3</b></p>  <p><input type="text"/></p>	<p><b>4</b></p>  <p><input type="text"/></p>
<p><b>5</b></p>  <p><input type="text"/></p>	<p><b>6</b></p>  <p><input type="text"/></p>
<p><b>7</b></p>  <p><input type="text"/></p>	<p><b>8</b></p>  <p><input type="text"/></p>
<p><b>9</b></p>  <p><input type="text"/></p>	<p><b>10</b></p>  <p><input type="text"/></p>

Drawing courtesy of Anta publications

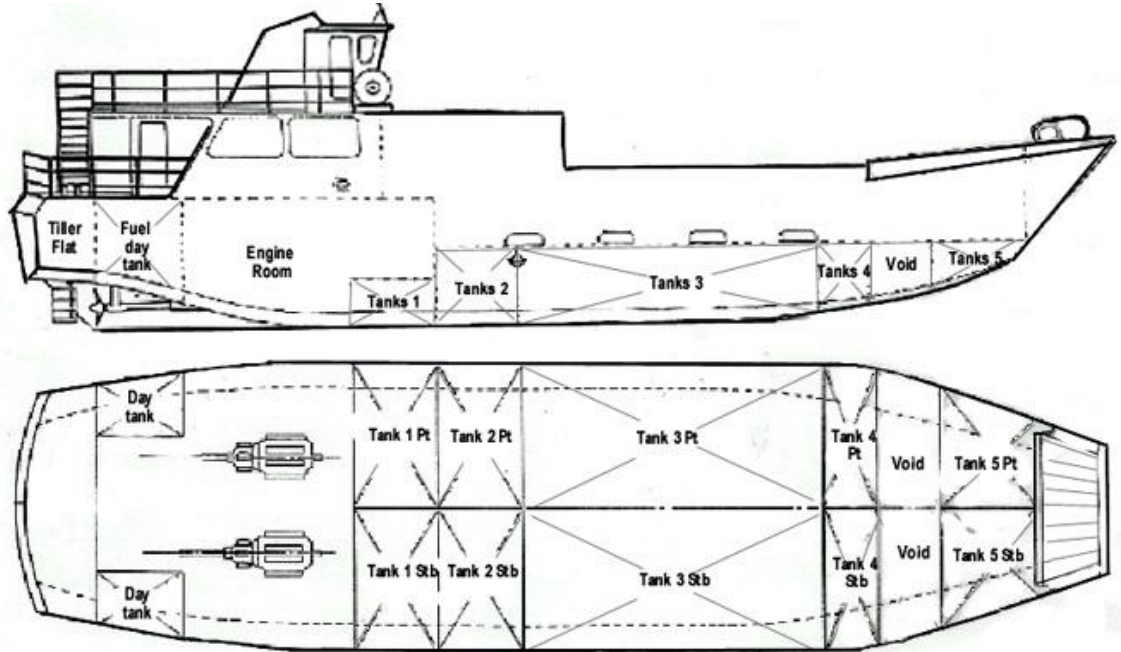
Answers reading a micrometer - as below:

1.  $12.5\text{mm} + 0.19\text{mm} = 12.69\text{mm}$
3.  $1.5\text{mm} + 0.39\text{mm} = 1.89\text{mm}$
5.  $0.5\text{mm} + 0\text{mm} = 0.5\text{mm}$
7.  $19.5\text{mm} + 0.05\text{mm} = 19.55\text{mm}$
9.  $21\text{mm} + 0.14\text{mm} = 21.14\text{mm}$

2.  $23.5\text{mm} + 0.49 = 23.99\text{mm}$
4.  $5.5\text{mm} + 0\text{mm} = 5.5\text{mm}$
6.  $7.0\text{mm} + 0.22\text{mm} = 7.22\text{mm}$
8.  $2.5\text{mm} + 0.25\text{mm} = 2.75\text{mm}$
10.  $9\text{mm} + 0.10\text{mm} = 9.10\text{mm}$

## Chapter 4: Shape, size and capacity of tanks

Tanks store fuel or water and can provide a second skin to increase watertight spaces. Below are day tanks are fitted from which fuel is gravity fed to the motors. Port and starboard tanks 1 are for the fuel oil needs of the passage, and are regularly pumped to press up (fill) the day tanks. Tanks 2-3 are used for ballast or fresh water cargo, and tanks 4 & 5 are for oil cargo. The latter tanks are separated by a void that can be filled with water to limit the spread of fire (a coffer dam).



Sounding rods or sight gauges enable tank capacity to be measured. However, measurements at sea are inaccurate due to the rolling and pitching of the vessel. Using the vessel's plans of tanks and spaces and standard formulas enables the condition of loading of each tank can be calculated.

### 4.1 Definitions

*Length* is the longer horizontal measure.

*Width* is the shorter horizontal measure.

*Height* is the vertical measure.

*Area* is the space occupied by a shape, calculated by multiplying length and width.

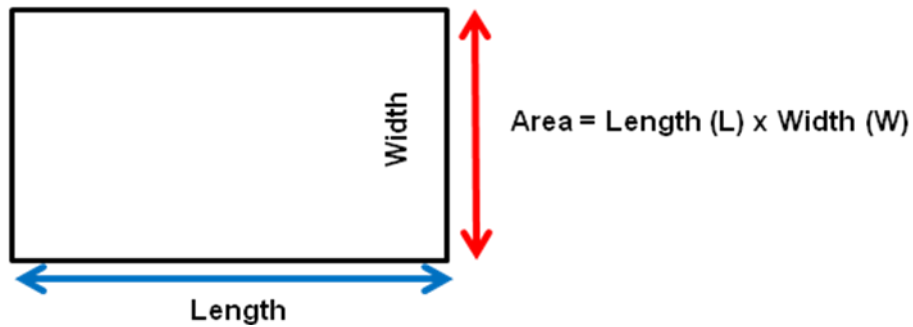
*Perimeter* is the distance around the outside (boundary) of a shape. It is calculated by adding all lengths and all widths.

*Volume* is the holding capacity, calculated by multiplying length, width and height.

## 4.2 Areas of Common Shapes:

Area is the measurement of the footprint for a two dimensional object.

**Rectangles** - The area is measured by multiplying the Length by the Width.

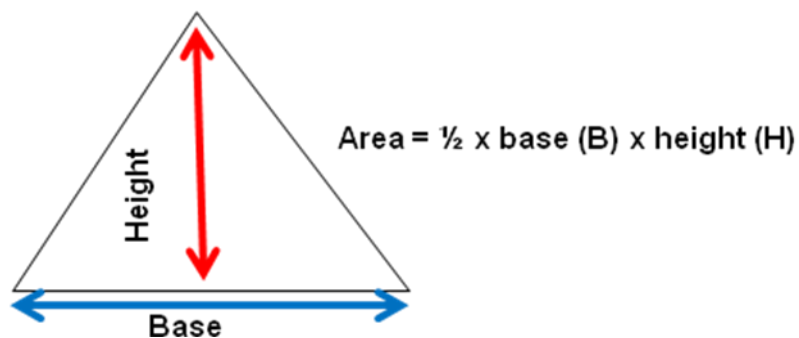


*Example:*

Find the area of a rectangle 10.2 metres long and 6.0 metres wide.

$$\begin{aligned}\text{Area} &= L \times W \\ &= 10.2 \times 6.0 \\ &= 61.2 \text{ m}^2 \text{ (square metres)}\end{aligned}$$

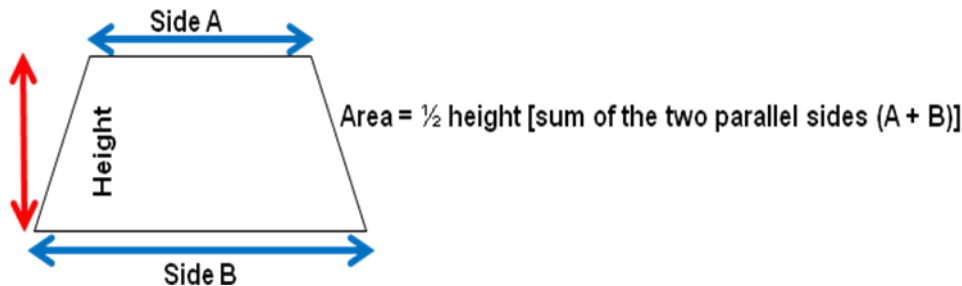
**Triangles** -The area of a triangle is calculated by multiplying half of the base of the triangle by the height of the triangle. Or equivalently, the base can be multiplied by the height and the result then divided by two.



*Example:* What is the area of a triangle with a base of 3.8 m and 1.1 m high?

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times B \times H \\ &= \frac{1}{2} \times 3.8 \times 1.1 \\ &= 2.09 \text{ m}^2 \text{ (square metres)}\end{aligned}$$

**Trapeziums** - A trapezium is a four sided figure that has only two parallel sides. Its area is calculated by multiplying half its height by the sum of both parallel sides. Where **A** & **B** are the parallel sides and **H** is the perpendicular (shortest) distance between them, the height. Note: Do not measure up one of the sides.

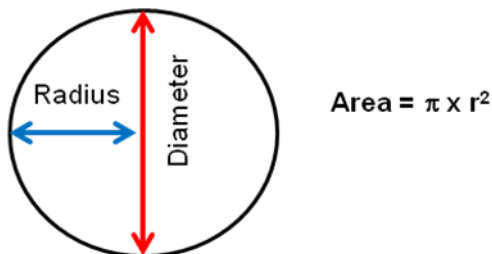


*Example:*

What is the area of a trapezium having parallel sides of 2.12 m and 3.1 m which are 1.2 m apart.

$$\begin{aligned}
 \text{Area} &= \frac{1}{2} \times (A + B) \times H \\
 &= \frac{1}{2} \times (2.12 + 3.1) \times 1.2 \\
 &= \frac{1}{2} \times (5.22) \times 1.2 \\
 &= 3.132 \text{ m}^2
 \end{aligned}$$

**Circles** - The area of a circle is given by using the formula:



Where  $\pi$  or pi = approximately 3.14.  
r or radius = half of the diameter of a circle.

*Example:*

Find the area of a 2.6 cm diameter circle. Give your answer to 2 decimal places.

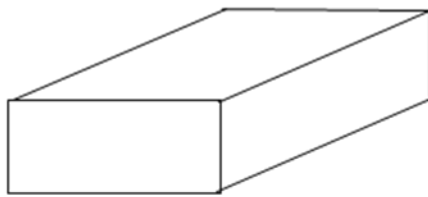
$$\begin{aligned}
 \text{Area} &= \pi \times (\frac{1}{2} \times 2.6)^2 \\
 &= \pi \times 1.3^2 \\
 &= 5.309291585^2 \\
 &= 5.31 \text{ cms}^2
 \end{aligned}$$

An alternative formula of  $\text{Area} = \frac{\pi}{4} \times \text{diameter}^2$  can be also used.

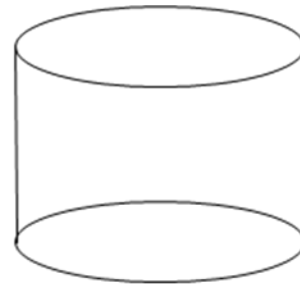
### 4.3 Volumes of tanks

Volume is the capacity measurement for three dimensional objects. Tanks can be considered to be “regular” or “irregular” in shape:

#### Regular shaped tanks:

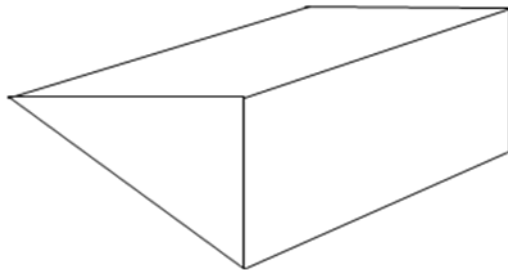


Rectangular Tanks

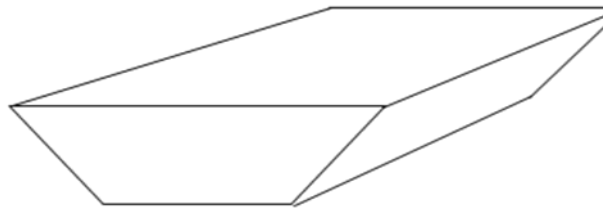


Cylindrical Tanks

#### Irregular shaped tanks:

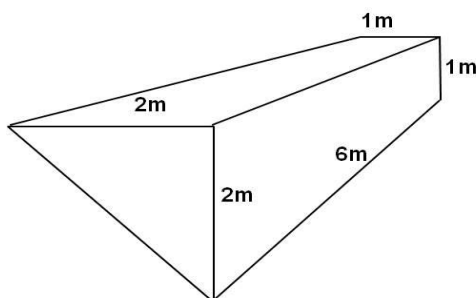


Triangular Tank

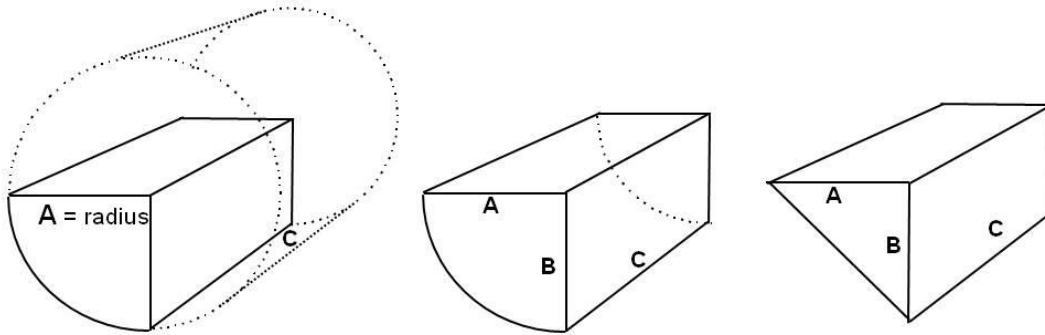


Trapezoidal Tank

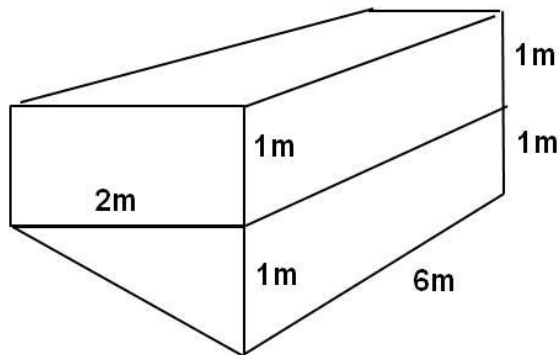
Tanks that taper also fit into this category.



In practical situations you may need to make calculations based on an approximate shape. For example, this curved tank can be approximated as a triangular tank or a quarter of a cylinder depending on the lengths of **A** and **B** and the curvature.



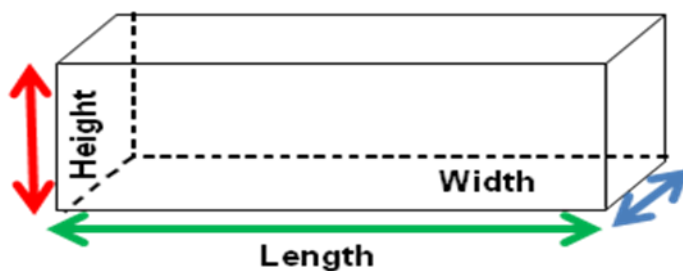
Alternatively, tanks may be considered as composite shapes and the capacity of section each calculated separately. For instance, the tank below is calculated as the composite of a rectangular top section added to the triangular bottom section to give the overall tank volume.



### **Rectangular Tanks:**

To calculate the volume (and capacity) of rectangular tanks the formula is Length multiplied by the Width multiplied by the Height of the tank.

$$\text{Volume} = \text{Length (L)} \times \text{Width (W)} \times \text{Height (H)}$$



*Example:*

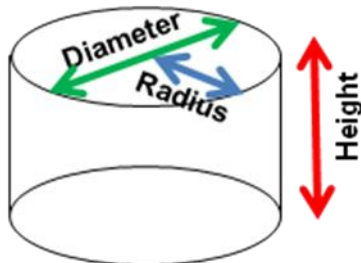
A tank is 3.1 m long, 2.24 m wide and 1.1 m deep, what is the volume.

$$\begin{aligned} \text{Volume} &= L \times W \times H \\ &= 3.1 \times 2.24 \times 1.1 \\ &= 7.6384 \text{ m}^3 \\ &= 7.64 \text{ m}^3 (\text{in cubic metres to 2 decimal places}) \end{aligned}$$

### ***Cylindrical Tanks:***

The volume of a cylindrical tank is measured by multiplying the area of the circle by the height or length of the tank.

$$\text{Volume} = \pi \times \text{radius } (r)^2 \times \text{height } (H)$$



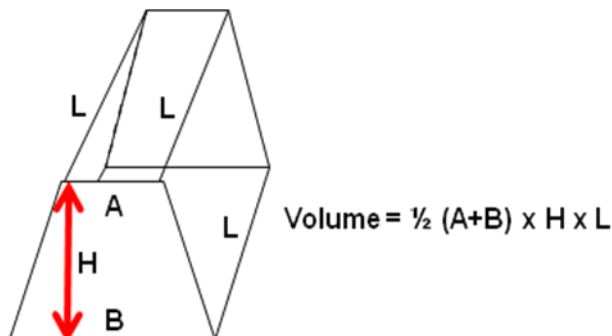
*Example:*

A cylinder has a circular base of 1.8m in diameter and stands 2.2m high. What is the capacity of the cylinder?

$$\begin{aligned}\text{Volume} &= \pi r^2 \times h \\ &= \pi \times 0.9^2 \times 2.2 \\ &= 5.595 \text{ m}^3\end{aligned}$$

### ***Trapezoidal tanks:***

Given the shape of some vessels and the limited space available below decks, it is often necessary to make fuel tanks in an irregular shape.



The area of a trapezium is calculated by multiplying half its height **H** by the sum of the two parallel sides **A** and **B**.

$$\text{Area} = \frac{1}{2}(A+B) \times H \times L$$

Once you have calculated the area of the side ends, you can calculate the volume of the tank by multiplying it by the length **L**.

*Example:*

Referring to the above shape, calculate the volume if the dimensions of the tank are:

$$A = 1.5 \quad B = 3 \quad H = 2 \quad L = 4$$

$$\text{Area} = \frac{1}{2} (A+B) \times H \times L$$

$$= \frac{1}{2} \times (1.5 + 3) \times 2 \times 4$$

$$= 4.5 \times 4$$

$$= 18 \text{ m}^3 \text{ (the tank has a volume of 18 cubic metres)}$$

## Measuring tank content

Four methods are available to measure the contents of a tank.

Gauges

Sight glass

Sounding

Ullage

Sounding <i>Depth of fuel</i>	Ullage <i>Depth to fuel</i>	Volume of fuel	SG 0.8	Weight of fuel
0.0 m	1.4 m	0.0 m <sup>3</sup> = 0 ltr	x 0.8	
0.2 m	1.2 m	0.2 m <sup>3</sup> = 200 ltr		
0.4 m	1.0 m	0.4 m <sup>3</sup> = 400 ltr		
0.6 m	0.8 m	0.6 m <sup>3</sup> = 600 ltr		
0.8 m	0.6 m	0.8 m <sup>3</sup> = 800 ltr		
1.0 m	0.4 m	1.0 m <sup>3</sup> = 1000 ltr		
1.2 m	0.2 m	1.2 m <sup>3</sup> = 1200 ltr		
1.4 m	0.0 m	1.4 m <sup>3</sup> = 1400 ltr		

## Check your progress

**a.** What is the volume of a tank measuring 7.1m long, 3m wide and 4500mm deep? Give your answer in cubic metres.

**b.** What is the volume of a ballast tank measuring 3080mm long, 1420mm wide and 64cm deep? Give your answer in m<sup>3</sup>. What is this tank's capacity in litres, to 2 decimal places. Remember that 1 Litre = 0.001 m<sup>3</sup> or 1000 Litres = 1 m<sup>3</sup>

**c.** What is the capacity of a cylindrical fresh water tank if it has a diameter of 6.2m and a length of 424.2cm? Give your answer to 2 decimal places.

**d.** What volume of fuel is in a cylindrical tank of 1.2 metres diameter and a height of 3 metres when it is half full?

**e.** What weight of fuel is in a cylindrical tank of 3 metres diameter and a height of 2 metres when there is an ullage taken of 400mm? Use a specific gravity for fuel of 0.8.



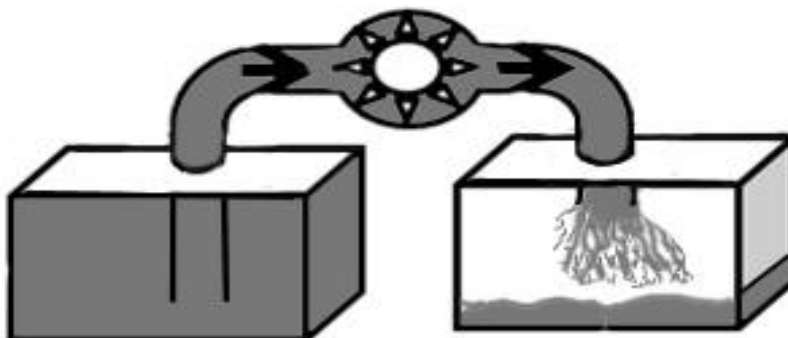
- f.** What weight of fuel is in a trapezoid tank of 2 metres bottom width and 2.4 metres top width, a height of 1.6 metre and a length of 4 metres? Use a specific gravity for fuel of 0.8.
- g.** What weight of fuel is used by a vessel travelling 300nm that maintains 10 knots with her single 200 kW engine consuming 125cc per kW per hour? Use SG 0.8.
- h.** Allowing in the above case for a 25% reserve, what weight of additional fuel would be prudent to load?

**Answers to check your progress:**

- a.** 95.85 m<sup>3</sup>
- b.** 2.799 m<sup>3</sup> 2799 litres
- c.** 12.80 m<sup>3</sup> 12800 litres
- d.** 1.69 m<sup>3</sup> 1690 litres
- e.** 11.3 m<sup>3</sup> 11304 litres x 0.8 = 9043.2 kilos = 9.043 tonnes
- f.** 14.8 m<sup>3</sup> 14800 litres x 0.8 = 11264 kilos = 11.264 tonnes
- g.** 750 litres x 0.8 = 600 kilos = 0.6 tonnes
- h.** An additional 150 kilos = 0.15 tonnes

## 4.4 Pumps and flow rate capacity

Pumping calculations are used when transferring or bunkering fuel. The time needed, the volume to move and the flow rate of the pump must be known. *Flow rate* is how quickly a pump transfers a volume of liquid. For calculating, the flow rate is found with the pump's manufacturer specifications. If not available you can calibrate the pump by transferring a sample volume while recording the time taken.



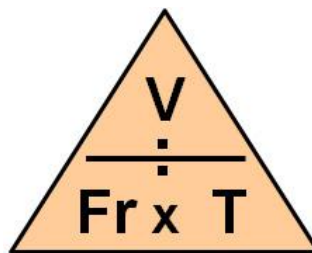
$$\text{Flow rate} = \frac{\text{Volume transferred}}{\text{Time taken}}$$

Where:

Fr = flow rate (litres/minute)

V = volume of liquid (litres)

T = time (minutes)



*Example 1. Flow Rate:*

What is the flow rate of a ballast pump is rated to pump out a 1000 litre tank in 20 minutes?

$$V \div T = Fr$$

$$1000 \div 20 = 50 \text{ litres per minute}$$

*Example 2. Time:*

How long will it take a 60 litres/minute pump to empty a 6000 litre tank?

$$V \div Fr = T$$

$$6000 \div 60 = 100 \text{ minutes} \div 60 = 1 \text{ hours } 40 \text{ minutes}$$

*Example 3. Volume:*

What volume of liquid can be transferred by a 15 litres/minute pump in 20 minutes?

$$Fr \times T = V$$

$$15 \times 20 = 300 \text{ litres}$$

*Example 4. Transfer time:*

Fuel is transferred from a main tank to a daily service tank at a rate of 7.5 litres/minute. If the pump operates for 12 minutes, how many litres will be pumped?

$$7.5 \times 12 = 90 \text{ litres}$$

If the daily service tank holds 210 litres, how much longer will be required to complete the transfer?

$$210 - 90 = 120 \text{ litres} \div 7.5 = 16 \text{ minutes}$$

## Transfers with multiple pumps

The principles and formula from above apply when calculating for pumps operating in parallel with different flow rates.

### Example 1. Paired pumps:

How long will it take to empty a 2 cubic metres tank using a pair of pumps. Pump A has a flow rate of 6 litres/minute and pump B has a flow rate of 4 litres/minute.

**Pump A flow rate + pump B flow rate = 6 + 4 = 10 litres/minute**

$$V \div Fr = T$$

$$2000 \div 10 = 200 \text{ minutes} \div 60 = 3.333 \text{ hours} = 3 \text{ hours } 20 \text{ minutes}$$

### Example 2. Paired pumps:

An older pump (A) has the capacity of 4 litres/minute.

A new pump (B) has a capacity of 8 litres/minute. What are the transfer capacities when operating in parallel to fill a 960 litre tank?

$$\text{Pump A capacity} = 960 \div 4 = 240 \text{ minutes} = 4 \text{ hours}$$

$$\text{Pump B capacity} = 960 \div 8 = 120 \text{ minutes} = 2 \text{ hours}$$

$$\text{A + B capacities/hour} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4} \text{ of } 960 \text{ litres} = 75\% \text{ of tank}$$

How long will it take to fill the 960 litre tank?

**75% tank transfer took 60 minutes, therefore combined transfer rate is:**  
**(960 litres x 75%) ÷ 60 = 12 litres/minute**

$$\text{Pumping time} = 960 \div 12 = 80 \text{ minutes} = 1 \text{ hours } 20 \text{ minutes}$$

### Example 3. Paired pumps:

A ballast pump A can empty out a 20000 litre tank in 2 hours. A feed pump B can empty out the same tank in 4 hours. If a crew member used both pumps at the same time, how long would it take to empty the 20000 litre tank.

$$\text{Capacities per hour} = \frac{1}{2} (A) + \frac{1}{4} (B) = \frac{3}{4} \text{ (Both) of tank per hour}$$

$$\text{Total time required} = 1 \text{ hour } \frac{60}{60} \div \frac{3}{4} = 1\frac{1}{3} = 1 \text{ hour } 20 \text{ minutes}$$

### Example 4. Paired pumps:

Calculate the time to empty a 12,000 litre tank if the pump's flow rate is 3000 litres per hour for the first three hours but slows to 1500 litres per hour later.

$$3000 \text{ litres} \times 3 = 9000 \text{ litres in 3 hours}$$

$$12000 - 9000 = 3000 \div 1500 = 2 \text{ hours}$$

$$\text{Total time} = 3 + 2 = 5 \text{ hours to empty}$$

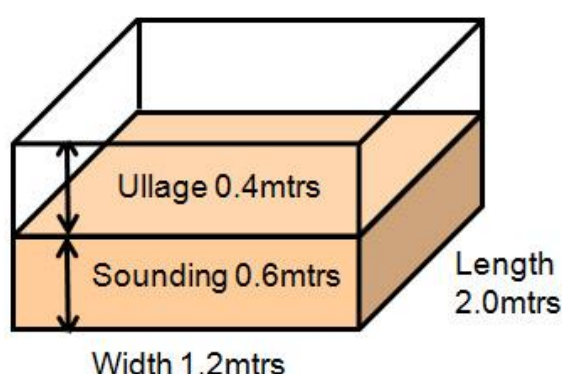
## Chapter 5: Calculating fuel requirements for a voyage

### 5.1 Endurance

*Endurance* is a term to describe how long a vessel is able to sustain itself before re-supply with the necessary manpower, stores, spare parts and above all, fuel to keep going and return. This section concentrates on estimations of fuel use.

#### Soundings and ullages

Due a vessel's movement tanks with float based fuel gauges are unreliable, so *sounding* or *ullages* are used instead. A sounding is a measure of the depth of liquid carried out with sounding rod (a graduated wooden stick). An ullage is a measure of the vacant tank (from top to fuel surface). Only for regular shaped tanks will the graduations on rods be evenly spaced. Calculations will be needed to prepare rods for irregular shaped tanks where volume change with depth (e.g. trapezoidal tanks).



Volume can be measured in cubic centimetres, litres or cubic metres or as below:

$$1000 \text{ cm}^3 = 1 \text{ litre} \quad \text{and} \quad 1000 \text{ litres} = 1 \text{ m}^3 \text{ or,}$$

$$1 \text{ m}^3 = 1000 \text{ litres} = 1,000,000 \text{ cm}^3$$

The theoretical amount of fuel required for a voyage can also be calculated from the engine's fuel consumption, vessel speed and distance to travel. Weather and loading conditions influence fuel consumption so allowance for these factors (an additional reserve) is usual when deciding how much fuel to load for a voyage.

#### Speed, distance and time

The formulas of speed, distance and time were discussed earlier in Chapter 3 Section 3.2. In short:

$$s = \frac{d}{t} \qquad d = s \times t \qquad t = \frac{d}{s}$$

Where:

**s** = speed (knots)    **d** = distance (nautical miles)    **t** = time (hours)

\*Note:        1852 metres = 1 nautical mile                    1 nautical mile/hour = 1 knot

*Example 1: Theoretical fuel consumption*

What is the theoretical hourly fuel consumption for a vessel if it travels at a speed of 10 knots on a 640 nautical mile voyage and consumed 320 litres of fuel.

$$d \div s = t$$

$$640nm \div 10 \text{ knots} = 64 \text{ hours}$$

$$320 \text{ litres} \div 64 \text{ hours} = 5 \text{ litres/hour}$$

*Example 2: Fuel used during a voyage*

A trawler has a fuel tank as above of 1 metre high, 2 metres long and 1.2 metres wide. After 40 hours steaming at 14 knots an ullage of 0.4 metres is read. How much fuel is left and how much further can the vessel steam with the remaining fuel.

$$\text{Total tank volume} = 1 \times 2 \times 1.2 = 2.4 \text{ m}^3 = 2400 \text{ litres}$$

$$\text{Volume used} = 0.4 \times 2 \times 1.2 = 0.96 \text{ m}^3 = 960 \text{ litres}$$

$$\text{Volume left} = 0.6 \times 2 \times 1.2 = 1.44 \text{ m}^3 = 1440 \text{ litres}$$

$$\text{Consumption} = 960 \text{ litres} \div 40 \text{ hours} = 24 \text{ litres/hour}$$

$$\text{Hours remaining} = (1440 \text{ litres} \div 24) = 60 \text{ hours}$$

$$\text{Distance in miles} = 60 \times 14 = 840 \text{ nautical miles additional}$$

*Example 3: Fuel for a voyage*

A vessel has a regular shaped fuel tank measuring 0.8m high, 3.4m long and 2.6m wide. What is its volume?

$$0.8m \times 3.4m \times 2.6m = 7.72 \text{ m}^3$$

If the tank is 60% full, what will be the ullage?

$$100\% - 60\% = 40\% \times 0.8 = 0.32m$$

If the tank is 60% full, what is the volume of remaining fuel?

$$60\% \times 7.72 = 4.632m = 4632 \text{ litres}$$

If the vessel's 400 kW engine uses 0.16 litres/kW/hour to attain 10 kts, how far can it travel with the remaining 60% of fuel?

$$400 \times 0.16 = 64 \text{ litres/hour} \quad 4632 \div 64 = 72.375 \text{ hours} \quad 10 \times 72.375 = 723.75 \text{ nm}$$

**Example 4: Fuel for a voyage**

A trawler has a cylindrical fuel tank of 3 metres high with a diameter of 2.6 metres. If the vessel's 250 kW engine consumes 0.3 litres/kW/hour to attain 15 kts, how far can it travel with a full tank? Allow a 15% reserve.

$$\begin{aligned}\text{Tank volume} &= \pi r^2 \times h \\ &= \pi \times 1.3^2 \times 3 \\ &= 15.92994 \text{ m}^3 = 15929.94 \text{ litres}\end{aligned}$$

$$\text{Fuel consumption at 15 kts} = 250 \times 0.3 \text{ litres/hour} = 75 \text{ litres/hour}$$

$$\begin{aligned}15\% \text{ reserve} &= 75 \times 15\% = 11.25 \text{ hours} \\ &= 75 - 11.25 = 63.75 \text{ hours capability}\end{aligned}$$

$$\text{Distance in miles} = 63.75 \times 15 = 956.25 \text{ nautical miles}$$

**Example 5 Steaming Time Remaining**

A vessel steams at 12 knots and consumes 15 litres/hour. If the full tank held 900 litres calculate the steaming time remaining if half the tank is used.

$$900 \text{ litres} \div 2 = 450 \text{ litres.}$$

$$450 \text{ litres} \div 15 \text{ litres/hour} = 30 \text{ hours of steaming time left.}$$

**Check you progress with speed, distance and fuel consumption:**

**a.** In a vessel that travels at 10 knots consuming 12 litres per hour, how much fuel is required to travel 360 nm before allowing for reserves?

**b.** In a vessel that travels at 12 knots with twin motors consuming 15 litres per hour each, how much fuel is required to travel 400 nm, allowing for reserves of 20%?

**c.** How far can a vessel travel with 1000 litres of fuel aboard with a single motor consuming 20 litres an hour to give a constant speed of 18 knots?

**d.** How far can a vessel travel with 2000 litres of fuel aboard with twin motors consuming 10 litres an hour each to give a constant speed of 10 knots?

**e.** A vessel has 1000 litres of fuel aboard. Its single motor burns 20 litres an hour at 10 knots. How much extra fuel is required to steam 2000 nm without reserves?

**f.** A vessel has 560 litres of fuel aboard. Each of its twin motors burn 8 litres an hour when at 6 knots. How much extra fuel is required to steam 600 nm?

**Answers to speed, distance and fuel consumption:**

**a.**  $360 \div 10 = 36 \text{ hours}$        $36 \times 12 = 432 \text{ litres}$

**b.**  $400 \times 20\% = 80 \text{ nm}$        $480 \div 12 = 40$        $40 \times 15 = 600 \text{ litres}$

**c.**  $1000 \div 20 = 50 \text{ hours}$        $50 \div 18 = 900 \text{ nm}$

**d.**  $10 \times 2 = 20 \text{ ltr/hour}$        $2000 \div 20 = 100 \text{ hours}$        $100 \times 10 = 1000 \text{ nm}$

**e.**  $1000 \div 20 = 50 \text{ hours}$        $50 \times 10 = 500 \text{ nm}$        $2000\text{nm} \div 500\text{nm} = 0.25 \text{ of voyage}$

$1000 \times 3 = 3000 \text{ additional litres required}$

**f.**  $560 \div 8 = 70 \text{ hours}$        $70 \times 6 = 420 \text{ nm}$        $600\text{nm} - 420\text{nm} = 180 \text{ remaining}$

$180 \div 6 = 30 \text{ additional hours required}$        $30 \text{ hrs} \times 8 \text{ ltrs} = 240 \text{ additional litres}$

## 5.2 Specific fuel consumption and fuel coefficients

Fuel quantity can be expressed as a volume or a mass (\*weight). To convert between the two we use the fuel's *Density* (see Chapter 3 Sect. 3.2.) In short, *Relative Density* or *Specific Gravity* of a substance is a comparison of the mass of a volume of one substance to the mass of an equal volume of pure water. S.G is the mass compared to pure water valued as 1 (it is a ratio so it has no units). *R.D.* is expressed as a percentage of pure water's mass.

$$\text{S.G. ratio} = \frac{\text{mass of substance}}{\text{mass of fresh water}} = \text{R.D. percentage}$$

Depending on the grade, diesel fuel has a S.G of approximately 0.84 or R.D. of 84% that of the same volume of water. A quantity of fuel can be expressed by:

$$\text{Mass (M)} = \text{Volume} \times \text{S.G. or R.D.}$$

or

$$\text{Volume (V)} = \frac{\text{Mass}}{\text{S.G. or R.D.}}$$

Remember to use consistent units (as below) within your calculations

Tonnes	(t)	and	cubic metres	(m <sup>3</sup> )	
Kilos	(kg)	and	Litres	(ltr)	ratio or percentage
Grams	(gms)	and	cubic centimetres	(cc)	

**\*Note** - although equivalent for sea level calculations, technically *weight* unlike *mass* is an object's measured downward pressure at a specified gravitational locality.

*Example 1: Tonnes and cubic metres*

What is the mass of a 2 m<sup>3</sup> tank full of 0.8 S.G. fuel?

$$M = V \times SG = 2 \text{ m}^3 \times 0.8 = 1.6 \text{ tonnes}$$

*Example 2: Litres and kilograms*

What is the mass of a 2000 litre tank full of 80% R.D. fuel?

$$M = V \times RD = 2000 \times 80\% = 1600 \text{ kilograms}$$

*Example 3: Kilograms and litres*

What is the volume of a 1600 kilo tank full of 0.8 S.G. fuel?

$$V = \frac{M}{SG} = \frac{1600}{0.8} = 2000 \text{ litres}$$

*Example 4: Grams and cubic centimetres*

What is the volume of a 1600 gram tank full of 0.8 S.G. fuel?

$$V = \frac{M}{SG} = \frac{1600}{0.8} = 2000 \text{ cc}$$



### Specific fuel consumption and efficiency

Engine manufacturers use *brake specific fuel consumption (BSFC)* as a measure of the fuel efficiency of a rotational or shaft driving engine. They measure the mass of fuel consumed compared to each kilowatt of power that an engine generates in order to rate performance. Expressed as grams used per kilowatt per second (gms/kW/s), they will use the formulas below:

$$\text{Per Second} \quad \text{BSFC (gms/kW/s)} = \frac{r}{P}$$

where:

$r$	=	fuel consumption rate-	in grams per second (gms/s)
$P$	=	power produced-	in watts where $P = t \times w$
$w$	=	engine speed-	in radians* per second (rad/s)
$t$	=	engine torque-	in newton meters* (N/m)

\*Note (see Chapter 3 Sect. 3.2)

The values of  $r$ ,  $w$ , and  $t$  are measured from a running test engine to give a BSFC in grams per joule (gms/J).

The marine engineer commonly refers to *specific fuel consumption* as grams per kilowatt per hour (gms/kW/h) or pounds per \*HP per hour (lb/hp/h) that may convert as in the formulas below:

<b>Per Hour</b>	<b>BSFC (gms/kW/h)</b>	<b>=</b>	<b>BSFC (gms/J) x (3.6x106)</b>
Metric to Imp.	<b>BSFC (g/kW/h)</b>	<b>=</b>	<b>BSFC (lb/hp/h) x 608.277</b>
Imp. to Metric	<b>BSFC (lb/hp/h)</b>	<b>=</b>	<b>BSFC (g/kW/h) x 0.001644</b>

\*Note American and older engines may be rated in Horse Power (H.P.). To convert to kW multiply by 0.74.

For a diesel engine of 1000kW on load, a typical specific fuel consumption is around 200 gms/kW/hr. This is found in the manufacturer's engine specification or may be observed through the vessels fuel records. Monitoring the constancy of fuel consumption will be a guide to the engines continuing optimum performance.

Calculating fuel efficiency of an engine requires the energy potential of the fuel, called its *heating value*. The lower heating values for fuels used in small marine high speed internal combustion engines include:

Petrol = 0.0122225 gms/kW/h (18,917 BTU/lb)

Diesel = 0.0119531 gms/kW/h (18,500 BTU/lb)

#### Example: SPC

An engine using SG 0.8 fuel consumes 100 litres per hour to maintain 400 kW output. What is the specific fuel consumption and the engines efficiency? *Answer:*

400 kW is generated using 100 litres per hour

$400 \div 100 = 4\text{kW}$  is generated using 1 litres per hour

1 litre of fuel weighs 0.8 kilos ( $0.8 \times 1000 = 800$  grams)

$800 \div 4 = \mathbf{200 \text{ gms/kW/hour}}$

The 200 gms/kW/hr diesel engine's efficiency =  $1 \div (\text{SFC} \times 0.0119531) = \mathbf{41.8\%}$

### Calculations using fuel coefficients

Internal combustion engines' thermal efficiency depends on load but generally peaks around 70% of maximum load. Fuel consumption (FC) can be estimated by using the vessel's displacement and speed (the assumed load) with a fuel coefficient. The fuel coefficient is an indication of the vessel's engine efficiency - the higher the fuel coefficient, the higher the engine efficiency. The coefficient will be provided with the naval architect's specifications. The calculation remains indicative only as sea conditions, hull form and loading condition will have an appreciable effect on actual performance. Estimated fuel consumption is calculated with the formula below:

$$\begin{aligned}\text{Fuel Consumption} &= \frac{\text{Vessel displacement}^{2/3} \text{ (tonnes)} \times \text{Speed}^3 \text{ (knots)}}{\text{Fuel coefficient}} \\ &= \Delta^{2/3} \times S^3\end{aligned}$$

\*Note See [Chapter 1.4 for Scientific notation - exponents and roots](#)

#### Example:

A 250 tonnes vessel with 436kW engine has a fuel coefficient of 58,226 at 15 knots.

What is the estimated daily fuel consumption at the rated speed?

How far will it travel in 24 hours?

What is the average SFC?

What is the efficiency of the engine?

#### Answers:

$$\text{Daily FC} = \frac{\Delta^{2/3} \times S^3}{\text{Fuel coefficient}} = \frac{250^{2/3} \times 15^3}{58,226} = \frac{39.68 \times 3375}{58,226} = \mathbf{2.3 \text{ tonnes}}$$

$$\text{Dist.} = 15 \text{ kts} \times 24 \text{ hrs} = \mathbf{360 \text{ nm}}$$

$$\text{SFC} = \frac{2.3 \text{ t} \div 24 \text{ hr}}{436 \text{ kW}} = \frac{0.958333 \text{ t/hr}}{436 \text{ kW}} = \frac{95833.3 \text{ gms/hr}}{436 \text{ kW}} = \mathbf{220 \text{ gms/kW/hr}}$$

$$\text{Eff.} = 1 \div (\text{SFC} \times 0.0119531) = 1 \div (220 \times 0.0119531) = \mathbf{38\%}$$

#### Question 1:

A 500 tonne vessel with 899kW engine has a fuel coefficient of 75,000 at 17 knots.

What is the estimated daily fuel consumption at the rated speed?

How far will it travel in 24 hours?

What is the average SFC?

What is the efficiency of the engine?

**Answer 1:**

$$\text{Daily FC} = \frac{\Delta^{2/3} \times S^3}{\text{Fuel coefficient}} = \frac{500^{2/3} \times 17^3}{75,000} = \frac{62.99 \times 4913}{75,000} = \mathbf{4.13 \text{ tonnes}}$$

$$\text{Dist.} = 17 \text{ kts} \times 24 \text{ hrs} = \mathbf{408 \text{ nm}}$$

$$\text{SFC} = \frac{4.13 \text{ t} \div 24 \text{ hr}}{899 \text{ kW}} = \frac{0.1720833 \text{ t/hr}}{899 \text{ kW}} = \frac{172083.3 \text{ gms/hr}}{899 \text{ kW}} = \mathbf{191.42 \text{ gms/kW/hr}}$$

$$\text{Eff.} = 1 \div (\text{SFC} \times 0.0119531) = 1 \div (191.42 \times 0.0119531) = \mathbf{43\%}$$

**Question 2:**

A 200 tonnes vessel with 291 kW engine has a fuel coefficient of 53500 at 14 knots.

What is the estimated daily fuel consumption at the rated speed?

How long will a voyage 294 nm take?

If it used 1530 litres of 0.8 SG fuel during the voyage, what was the average SFC?

What was the efficiency of the engine?

**Answer 2:**

$$\text{Daily FC} = \frac{\Delta^{2/3} \times S^3}{\text{Fuel coefficient}} = \frac{200^{2/3} \times 14^3}{53,500} = \frac{34.20 \times 2744}{53,500} = \mathbf{1.75 \text{ tonnes}}$$

$$\text{Time} = 294 \text{ nm} \div 14 \text{ kts} = \mathbf{21 \text{ hrs}}$$

$$\text{Fuel} = 1530 \text{ ltr} \div 21 \text{ hrs} = 72.85 \text{ ltr/hr} \times 0.8 = 58.28 \text{ kilos/hr} = \mathbf{58280 \text{ gms/hour}}$$

$$\text{SFC} = \frac{58280 \text{ gms/hr}}{291 \text{ kW}} = \mathbf{200.27 \text{ gms/kW/hour}}$$

$$\text{Eff.} = 1 \div (\text{SFC} \times 0.0119531) = 1 \div (200.27 \times 0.0119531) = \mathbf{42\%}$$

## 5.3 Propeller pitch

### Propellers

Viewed from the stern of a vessel the shaft may be configured to turn in a clockwise direction (*right handed*) or in an anticlockwise direction (*left handed*). Due to the differential water density and aeration experienced by a propeller's uppermost submerged blades, a right handed prop will create a *paddle wheel effect*. This pushes the vessels head to port when ahead propulsion is first engaged and pushes the vessels stern to port when astern propulsion is first engaged. The inertia of a vessel accelerating from a stall (being stopped in the water) accentuates this effect.

With twin propped vessels the starboard prop is usually configured to be right handed and the port prop to be left handed. Some props may be set up as twins on the same shaft, counter rotating (*balanced props*) to negate this *paddle wheel* or *transverse thrust effect*.

A propeller is described by its number of blades, the direction it turns, its diameter and its pitch. The prop may have two, three or multiple blades. Pitch is the theoretical distance that a propeller would drill itself for each revolution through a solid medium (like a drill auger) and is determined by the blades' angle to the shaft. Operating at relatively high speeds in a liquid medium, the actual distance that the propeller drills through the water is less than the theoretical distance, this difference being termed *slip*. Additionally, a high speed prop excites the water around itself, both sucking air down from the surface and de-pressurising water at the blades tips to release air bubbles, in a process called *cavitation*. Cavitation reduces the local density of water around the prop and consequently the thrust with which it can drive the vessel forward.

A four bladed, right handed, 75 cm diameter x 150 cm pitch propeller shown below.



## Gearbox ratio and propeller action

Every engine has designed crankshaft speeds for idling, cruise and full speed. This is the number of revolutions made by the crankshaft every minute and is quoted as revolutions per minute (rpm). As well as providing forward propulsion, astern propulsion and disengagement (neutral) the gearbox reduces the engine speed by a fixed ratio before rotating the propeller shaft. Most small marine engines work more efficiently by turning at a higher speed than their propellers would. The gearbox reduces the drive speed from the engine through the shaft to the propeller to reduce the likelihood of *excitation* and *slip*.

For example, a gearbox with a ratio of 3 to 1 will revolve the propeller once for every three revolutions of the engine's crankshaft. If the engine speed is 2400 rpm and the gearbox ratio is 3:1, the propeller will rotate at 800 rpm.

$$\text{Gearbox ratio} = \frac{\text{Engine speed}}{\text{Propeller speed}} = \frac{2400}{800} = 3:1 \text{ Ratio}$$

The distance the propeller theoretically drills itself through water is directly related to its pitch and the speed it turns, as in the formula:

$$\text{Distance travelled (metres/minute)} = \text{prop pitch (metres)} \times \text{prop speed (rpm)}$$

We measure the speed of the vessel in nautical miles per hour (knots) and calculate the distance travelled from the hours travelled at that speed. With a *50 cm diameter x 75 cm pitch propeller*, each revolution theoretical would move the boat 0.75 mtrs forward. Calculating with 1852 mtrs per nautical mile and 350 revolutions per minute:

$$\begin{aligned} \text{Theoretical speed (knots)} &= (\text{Prop pitch} \times \text{prop revs} \times \text{time}) \div 1852 \\ &= (0.75 \text{ mtrs} \times 350 \text{ rpm} \times 60 \text{ mins}) \div 1852 = 8.5 \text{ knots} \end{aligned}$$

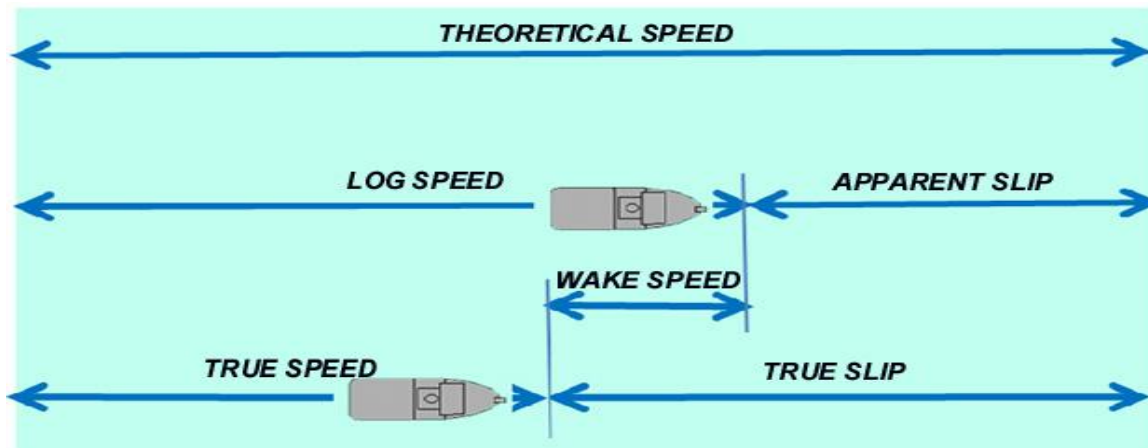
The theoretical speed is never reached due to the nature of water flow. The progress of the vessel through the water (*the apparent speed*) is often measured by a *log* (an impeller recording water flow past the hull). The log suffers error due to mis-calibration, water flow anomalies and the *wake* (the parcel of water that is dragged along with the vessel's hull). The log reads higher or lower than the *theoretical speed*, the difference being called the *apparent slip*. Above our 8.5 knots theoretical speed vessel only recorded 7.7 nm on its log.

$$\text{Theoretical speed} - \text{Log speed} = \text{Apparent slip (difference)}$$

$$8.5 \text{ kts} - 7.7 \text{ kts} = 0.8 \text{ knots positive apparent slip}$$

(Where log speed is less than theoretical speed it is called *positive apparent slip*.)

The *True speed* of the vessel between two points of water can be found from traditional or electronic position fixing adjusted for current and leeway. The difference between the *True speed* and the *Theoretical speed* is the *True Slip*. The relationship of the terms is shown below:



*Example 1:*

If a propeller speed is 400 rpm and the pitch is 1.2 metres, the vessel will theoretically travel a distance of 480 metres per minute, or 15.55 knots.

Theoretical speed (knots) = (Prop pitch x prop revs x time) ÷ 1852

$$400 \times 1.2 = 480 \times 60 = 28800 \div 1852 = 15.55 \text{ knots}$$

If the vessel's log read 12.55 nautical miles after an hour then propeller's apparent slip would be:

Theoretical speed - Log speed = Apparent slip

$$15.55 \text{ kts} - 12.55 \text{ kts} = 3.0 \text{ knots apparent slip}$$

If the vessel then determined that it had only progressed 10 miles during that hour (by position fixing) its true speed will be 10 knots. This means that the wake speed must be slowing the vessel by 2.55 knots.

True speed + wake speed + apparent slip = theoretical speed

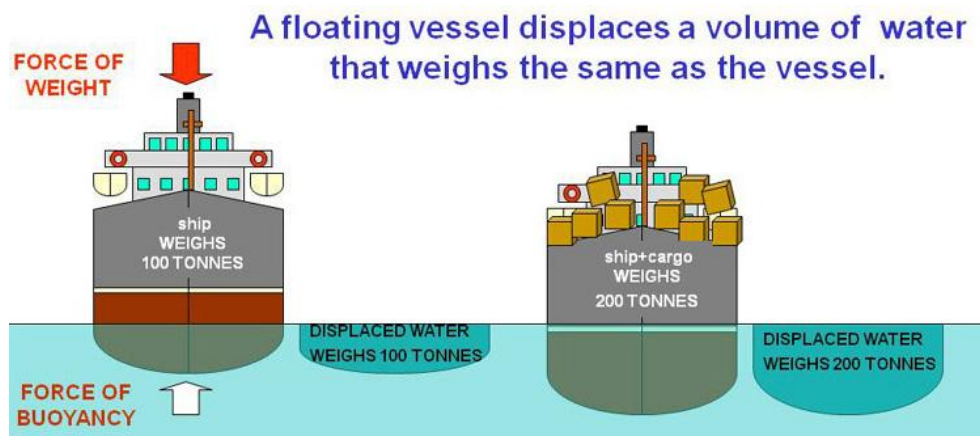
$$10 \text{ knots} + 2.55 \text{ knots} + 3 \text{ knots} = 15.55 \text{ knots theoretical speed}$$

## Chapter 6: Calculating a vessel's draught and trim

### 6.1 Displacement and buoyancy

#### **Displacement**

An immersed object pushes fluid aside (*displaces it*) and experiences an upward force called *buoyancy*. *Archimedes Principle* is that *objects wholly or partially immersed in fluid appear to suffer a loss in mass equal to the mass of fluid displaced*. That weight of water *displaced* is called the vessel's *displacement*. A vessel floats at a level where its weight and its buoyancy are balanced.



The ratio of the specific gravity (relative density) of a substance compares its weight to the weight of an equal volume of fresh water, expressed as:

$$\text{Specific Gravity} = \frac{\text{Weight of Substance}}{\text{Weight of Fresh Water}}$$

Example: One cubic metre of fresh water weighs one metric tonne - it has a density of 1.000 tonnes/m<sup>3</sup> (S.G. =1). Salt water, is heavier as one cubic metre of salt water weighs 1.025 tonnes – it has a density of 1.025 tonnes/m<sup>3</sup> (S.G. =1.025).

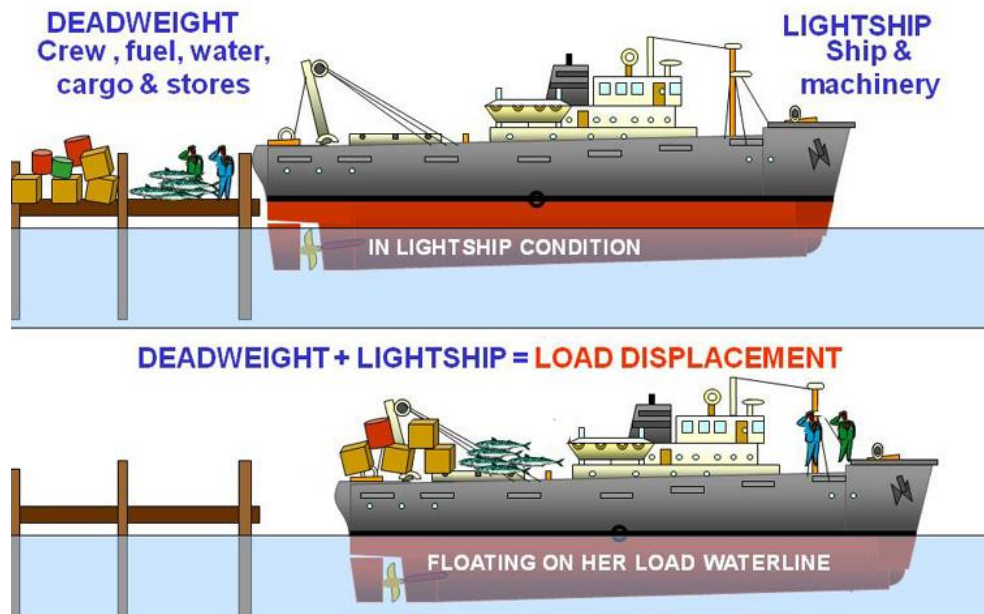
#### **Weight and buoyancy calculations**

If you lowered into fresh water a box that measured 1 cubic metre but contained a load of 3 tonnes it would experience an upward force of 1000 kg from the volume of water displaced and a downwards force of 3000 kg (a resultant force of 2000 kg downwards) - the box would sink.

If you lowered a box into fresh water that measured 5 cubic metre it could displace 5000 kg of fresh water. If the box weighed 3000 kg it would float in the water at a level where the forces are equal and opposite, that is, a force acting downwards of 3000 kg and 3000 kg upwards. However, there remains a potential to support another 2000 kg upwards force before the box will be fully immersed. The box would settle with 3 cubic metres below and 2 cubic metres above the water.

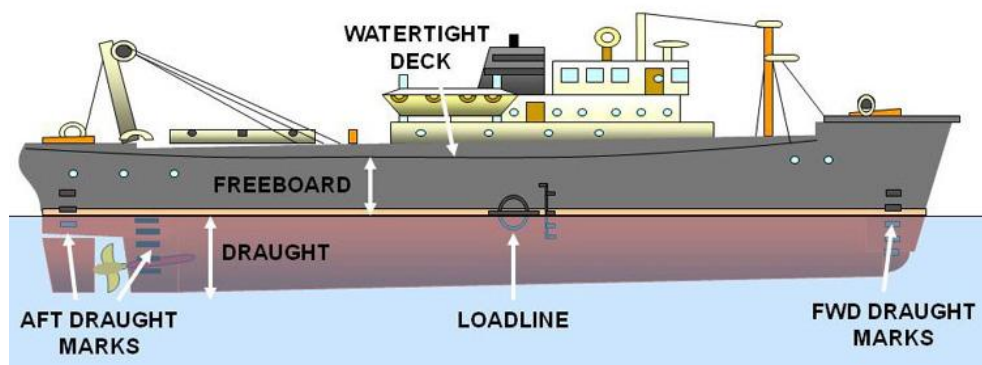
### **Lightship, deadweight and load displacement**

When fully loaded a vessel floats more deeply. This condition is called *load displacement*. When only the hull, superstructure, accommodation, machinery and lube oils/coolant is present it is said to float at *light displacement*. The difference between *load displacement* and *light displacement* is called *deadweight*. Fuel, fresh water, crew, gear, cargo or fish are all items of *deadweight*.



### **Draught and Freeboard**

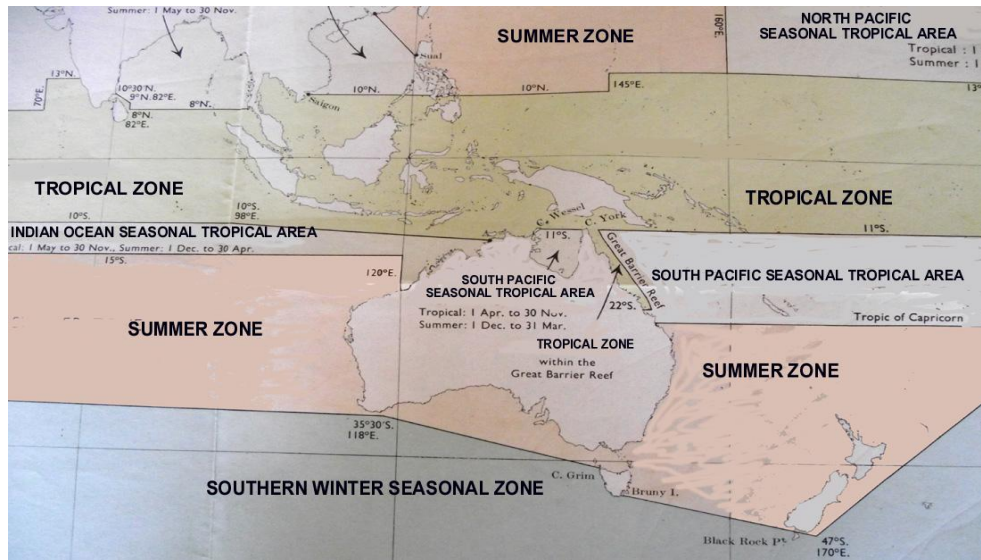
The distance from the vessel's bottom to the water surface is called its *draught*. The distance from the waterline to the lowest watertight deck is called *freeboard*. On loading, draught increases and freeboard diminishes. The numbers painted at the forward and after ends of the vessel (*draught marks*) enable reading its *condition of loading* and the *loadlines* (for vessels over 24 metres) painted around amidships enable reading its freeboard. The freeboard is a measure of the seaworthiness of the vessel, as it can drive through waves less than that height without the decks being awash. The vessel below floats on its *summer loadline* (or *plimsoll mark*).





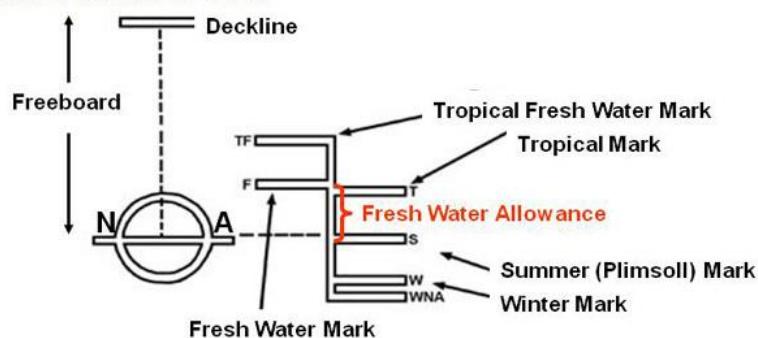
## Loadlines

The waterline (WL) describes the level at which the ship floats. The freeboard of a vessel is a measure of the amount of buoyancy remaining above the waterline, often referred to as *reserve buoyancy*. The freeboard is equal to the height of waves that the vessel can encounter before the decks become awash. The vessel above floats on its *summer loadline*, this being the loading mark for salt water voyages in temperate conditions. However, sea states vary greatly in the different oceans. The world is divided into summer and winter seasonal zones that anticipate typical roughness of local sea states and consequent need for additional freeboard.



Hence, loading to Winter mark (W) affords more freeboard than Tropical mark (T), and enables the vessel to retain greater buoyancy through rough seas. At no time may a vessel immerse the designated mark while on passage through a zone.

**To maintain adequate freeboard for each sea area's conditions the ship loads to the appropriate mark. The FWA enables the ship to load in the river but be on her marks at sea.**



## Fresh water allowance (FWA)

Many trading vessels load in the brackish or fresh water of river ports. Consequently they will float higher in the more buoyant salt water once at sea. In order to take on a full cargo but meet the marks at sea, the *fresh water allowance* (FWA) is applied.

FWA is the distance between the top of the Summer (S) line and the top of the Fresh (F) line. When a vessel loads in brackish water (*dock water*) its specific gravity must be tested with a hydrometer. The amount that the summer load line can be immersed is calculated as a percentage of the FWA equivalent to the different water densities.

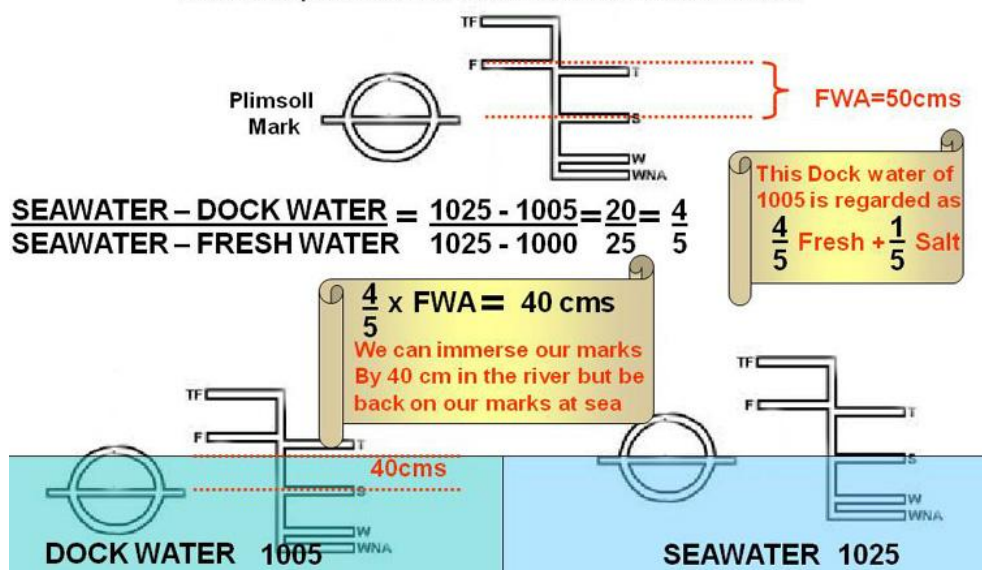
*Example:*

A vessel with a FWA of 50 cms loads in dock water of SG 1005. This water is only four fifths fresh so the vessel can only use four fifths of its FWA to float at the summer loadline out at sea. This calculates as 40 cms of its 50 cms FWA.

## VESSELS THAT LOAD IN PARTIALLY FRESH WATER

**A hydrometer is required to sample the dock water S.G.**

In the example dock water reads 1005 & the FWA is 50cms.



**Check your progress:**

A vessel with a FWA of 60 cms loads in dock water of S.G. 1015. How much can it immerse its summer loadline but be at that mark when it reaches the sea?

**Answer:**

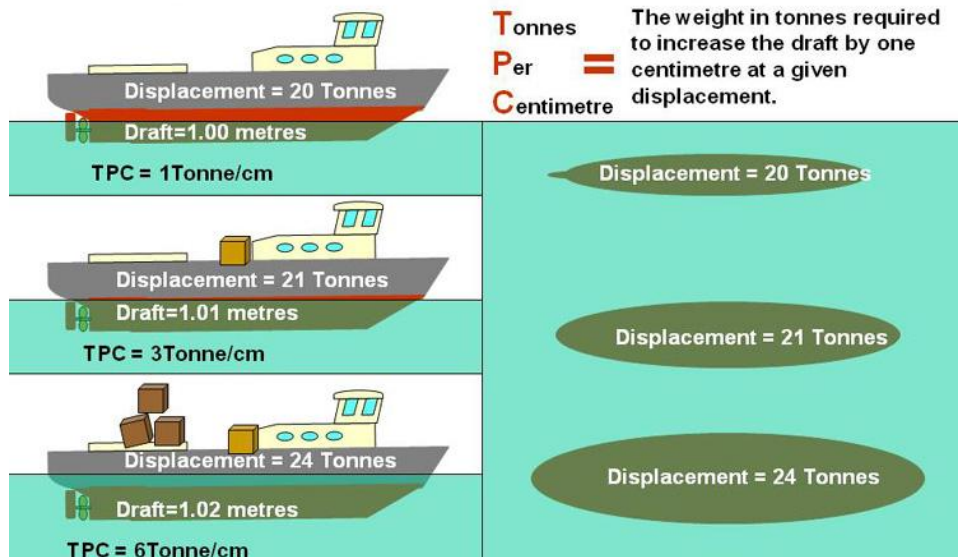
$$\frac{\text{SEA WATER}}{\text{SEA WATER}} - \frac{\text{DOCK WATER}}{\text{FRESHWATER}} = \frac{1025}{1025} - \frac{1015}{1000} = \frac{10}{25} = \frac{2}{5}$$

$$\frac{2}{5} \times 60 \text{ cm} = \frac{120}{5} = 24 \text{ cm over summer loadline}$$

### **Tonnes per Centimetre Immersion (TPC)**

As weight is loaded a vessel will sink lower in the water. As the vessel sinks lower its underwater shape usually alters requiring more or less weight to progressively change the draught another 1 cm. The weight that will sink the vessel 1 cm deeper is called the *tonnes per centimetre* immersion (TPC). This changing TPC for each displacement is found in the hydrostatic particulars of the vessels stability book.

**As the underwater shape of the hull changes on loading, the load in tonnes required to sink the vessel one centimetre (TPC) varies.**



The hydrostatic particulars for a vessel are tabulated as below. In the TPC column it is seen that at a draught of 2 mtrs and displacement 87.5 tonnes it requires 0.98 tonnes to sink the hull 1 cm. However at a draught of 3.25 mtrs and displacement 324.5 tonnes it requires 1.24 tonnes to sink the hull a further 1 cm.

### **Check your progress:**

Using a vessel's hydrostatic particulars as tabulated below,

- What is the TPC at 105.5 tonnes displacement?
- At a draught 2.9 mtrs, how much extra weight will increase the draught by 1 cm?
- At a draught 2.5 mtrs, how much extra weight will increase the draught to 3 mtrs?
- At a draught 2.4 mtrs, how much extra weight will increase the draught by 3 cms?

### **Answers:**

- 1.040 TPC
- 1.200 cm
- $204 - 145 = 59$  tonnes
- $(1.140 + 1.128) = 0.012 \times 5 =$  tonnes

### ***Hydrostatic particulars of a ship***

The hydrostatic particulars are unique to every ship and are prepared by the naval architect and the design team. This table shows examples of the displacement for various draughts on a level keel (parallel to the intended waterline markings).

<b>HYDROSTATIC DRAFT (m)</b>	<b>DISPLACEMENT (tonnes)</b>	<b>TPC</b>	<b>MCT 1cm (t - m)</b>	<b>LCF (m aft 0)</b>	<b>KM (m)</b>
2.00	87.5	0.980	0.885	- 0.295	4.47
2.05	93.5	1.000	0.945	- 0.380	4.41
2.10	101.0	1.022	1.000	- 0.465	4.35
2.15	105.5	1.040	1.060	- 0.555	4.29
2.20	111.2	1.062	1.115	- 0.650	4.24
2.25	116.8	1.078	1.172	- 0.750	4.20
2.30	122.0	1.098	1.235	- 0.860	4.15
2.35	127.5	1.115	1.292	- 0.955	4.11
2.40	133.2	1.128	1.340	- 1.010	4.07
2.45	139.0	1.140	1.380	- 1.045	4.03
2.50	145.0	1.150	1.445	- 1.065	4.00
2.55	150.5	1.160	1.430	- 1.070	3.97
2.60	156.5	1.168	1.438	- 1.072	3.94
2.65	162.5	1.172	1.465	- 1.065	3.91
2.70	168.0	1.180	1.480	- 1.060	3.89
2.75	174.0	1.185	1.500	- 1.050	3.87
2.80	180.0	1.190	1.515	- 1.042	3.85
2.85	186.0	1.195	1.532	- 2.038	3.84
2.90	192.0	1.200	1.550	- 1.030	3.82
2.95	198.0	1.208	1.568	- 1.022	3.81
3.00	204.0	1.212	1.585	- 1.015	3.81
3.05	210.0	1.218	1.605	- 1.008	3.80
3.10	216.2	1.220	1.622	- 0.995	3.80
3.15	222.2	1.225	1.640	- 0.988	3.79
3.20	228.2	1.230	1.660	- 0.978	3.79
3.25	324.5	1.240	1.670	- 0.968	3.78

*Hydrostatic draught* - the distance from a waterline to the lowest point of the keel, at a base line called (K).

*Displacement* - the weight of water displaced at a draught waterline.

*TPC* - the weight that will sink the vessel 1 cm deeper from its current waterline.

*MTC* - the moment required to change trim by 1 cm fore or aft. (MTC = tonnes x mtrs)

*LCG* - the position of longitudinal centre of gravity from the determined amidships.

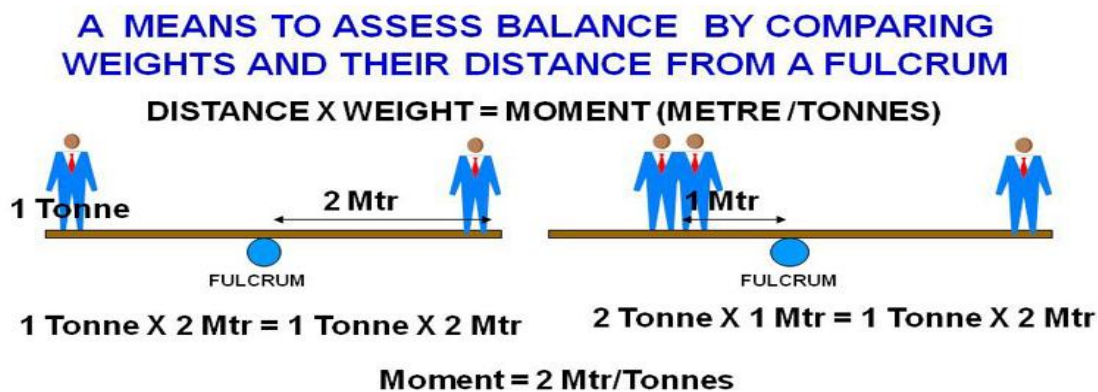
*KM* - the distance from the keel base line to the metacentre (M).



## 6.2 Moments and longitudinal trim

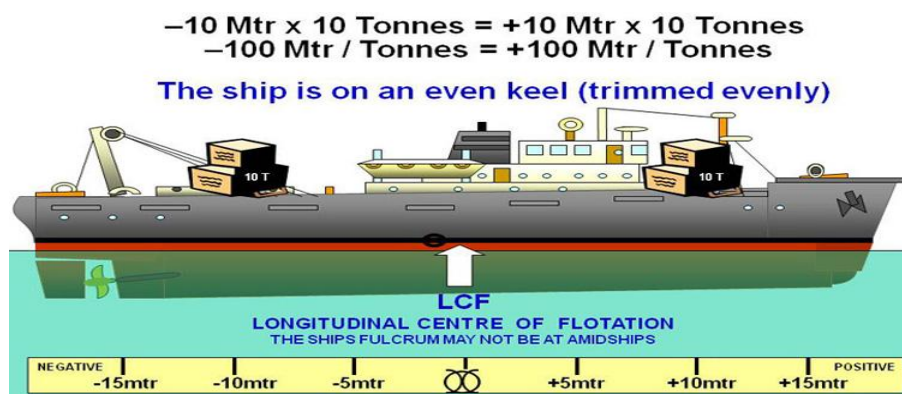
### Moments

*Moments* define rotation about a pivot point (*fulcrum*) and are used to assess balance. A *moment* is the product of a force and its distance from a *fulcrum* measured in metres/tonnes in stability calculations. A see-saw can be in balance with equal weights equally distant or by trading weight and distances for equal moments around the fulcrum. If the moments are equal the see-saw balances.

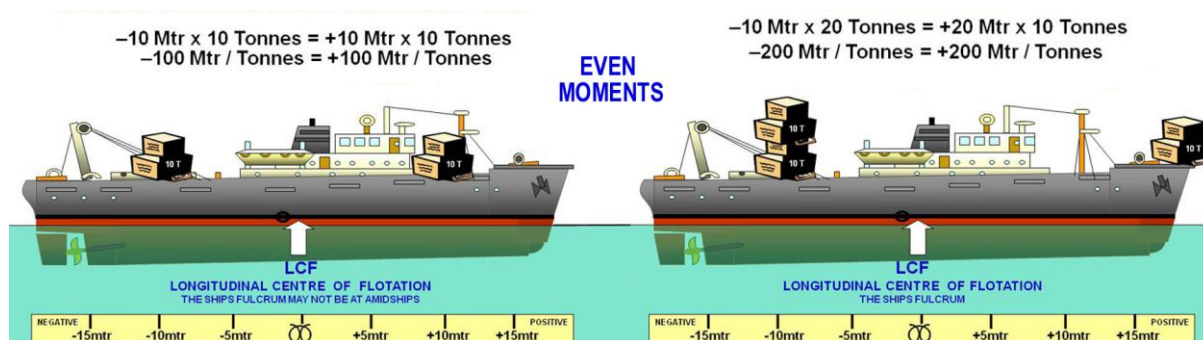


### Longitudinal trim

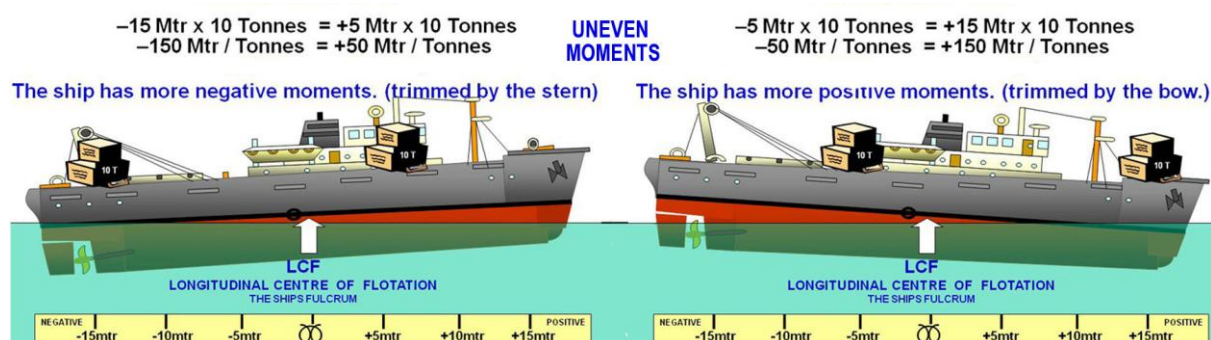
Like the see-saw, if equal weights as shown below are loaded at the same distance from the fulcrum, called the ship's *longitudinal centre of floatation* (LCF), the forward and aft draught will increase evenly. The vessel remains balanced and is said to be on an even keel. Note that the position of the LCF is not necessarily the same position as *amidships* or the *longitudinal centre of gravity* (LCG).



Balance can still be achieved if unequal weights are loaded provided that they are positioned at distances from the LCF that result in evenly distributed moments.



However, if weights are loaded resulting in unbalanced moments on each side of the LCF then the vessel will not sink evenly, change in draught will be unbalanced and the vessel will sink deeper at one end or the other.



The difference between the draught aft and the draught forward is called the trim and is expressed as:

$$\text{Trim} = \text{draught aft} - \text{draught forward}$$

If the vessel is *trimmed by the stern* (stern down) it is called *negative trim*, and if it is *trimmed by the head* (bow down) it is called *positive trim*.

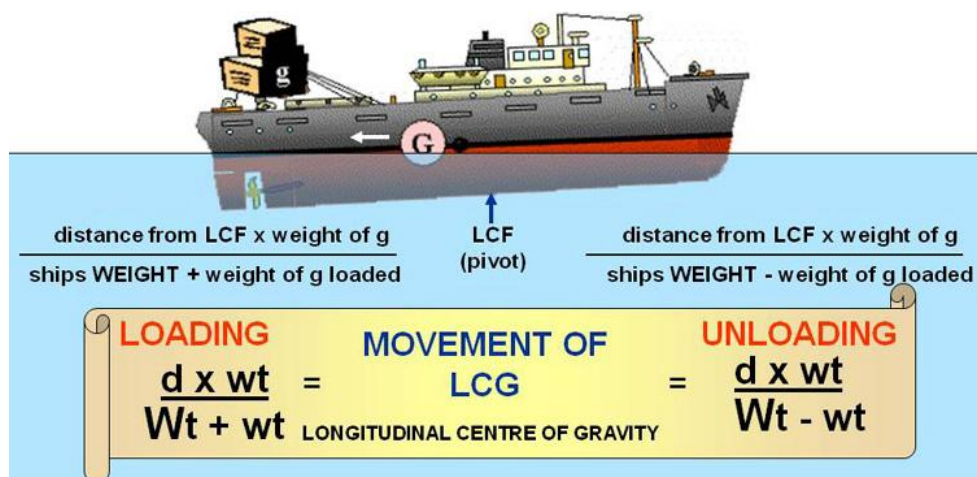
The terms negative and positive refer to the numeric values of distances aft or forward of the LCF. Negative and positive trim do not mean a good and bad condition. In fact, it is usually desirable to have your vessel negatively trimmed (by the stern) to increase reserve buoyancy forward, to ride more comfortably over head seas and be more responsive to a deeply immersed rudder and propeller. Excessive trim by the stern is not good as the vessel can be over responsive and less stable.

### **Change in longitudinal trim on loading**

The *longitudinal centre of gravity* (LCG) of a vessel is a position where all the combined weights of its structure and loading (all aft and forward moments) are considered to be centred.

If an additional weight (wt) is loaded the displacement of the vessel (Wt) is increased (Wt + wt). The balance is also changed by the moment (d x wt) and the vessels overall centre of longitudinal gravity moves towards the position of the weight loaded.

If a weight (wt) is unloaded from a vessel the overall displacement (Wt) is decreased by Wt - wt. The balance is changed by the moment (d x wt) and the centre of gravity (g) moves away from the position of the weight unloaded.



Added or removed weights are tabulated as negative and positive distances relative to the LCF. The sum of all the *longitudinal moments* are compared with the changed displacement to determine the changed centre of gravity (G), as tabulated below.

<b>Before loading</b>	<b>Wt tonnes</b>		<b>LCG metres</b>		<b>d x Wt metre/tonnes</b>
Lightship displacement	87.5	x	- 0.295	=	- 25.81
<b>After loading</b>	<b>wt</b>		<b>Distance</b>		<b>d x wt</b>
Loaded forward hold	2.0	x	+ 9.95	=	+ 19.90
Loaded after hold	4.0	x	- 20.61	=	- 82.44
<b>Load displacement</b>	<b>93.5 tonnes</b>				<b>- 88.35 mtr/tonnes</b>
Loaded LCG	=	$\frac{\text{Load longitudinal moments}}{\text{Load displacement}}$		=	$\frac{- 88.35}{93.50} = -0.945 \text{ metres}$

### Hydrostatic particulars

The vessel's Hydrostatic particulars also provide information about trim. Below it is found that the hydrostatic draught of the ship will increase from 2.00 metres to 2.05 metres.

<b>HYDROSTATIC DRAFT (m)</b>	<b>DISPLACEMENT (tonnes)</b>	<b>TPC</b>	<b>MCT 1cm (t - m)</b>	<b>LCF (m aft 0)</b>	<b>KM (m)</b>
2.00	87.5	0.980	0.885	- 0.295	4.47
2.05	93.5	1.000	0.945	- 0.380	4.41
2.10	101.0	1.022	1.000	- 0.465	4.35

At the new hydrostatic draught of 2.05, the Tonnes per centimetre column (TPC) lists that 1 tonne will be required to sink the vessel another 1 centimetre. Also, the Moments to change trim by 1 centimetre column (MCT 1cm) lists that a moment of 0.945 m/t will be required to alter the longitudinal trim by one centimetre.

### 6.3 Moments and transverse trim

The *transverse stability* of a vessel describes its ability to return to upright after heel. A vessel can experience a number of transverse states:

*Heel* is due to the external forces of wind and waves or the effects of making a turn. The angle to which a vessel will heel depends on how low its centre of gravity (G) is positioned. A vessel with low G will be stiff and return quickly to the upright, possibly straining its rigging and crew. A vessel with high G will be tender and return slowly to the upright, possibly succumbing to successive swells.

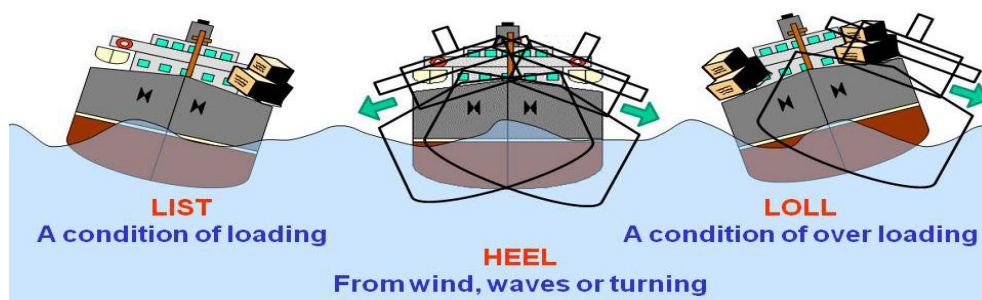
*Loll* is a condition too high a G, usually caused by top overloading. The vessel will sit equally well leaning to the port or starboard where a successive swell may roll it dangerously on its beam ends.

*Roll period* is a vessel's tendency, due to its design and/or loading, to roll rhythmically. *Synchronous rolling* can occur when the vessel's roll period and the swell length coincides, exaggerating the rolling. On recognising this condition the vessel must change its course or speed to change its time of encounter with the swell's period and reduce what may become dangerous rolling.

*Free surface effect* is caused by weights moving freely across the vessel during rolling. It takes its name from the exposed surface area of liquids in part filled tanks and can be considered as a virtual rise in G.

*Suspended weights* such as hauled fish nets or cargos significantly raise the position of G and reduce the vessels stability.

*List* is a condition of G displaced to the port or starboard caused by unequal loading or damage. The vessel will sit to the port or starboard and will handle badly.



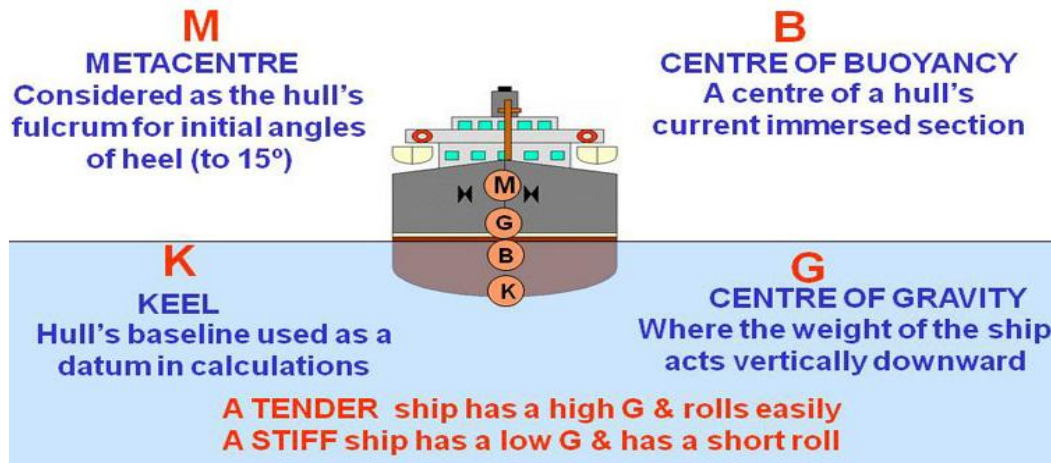
In order to describe how a vessel is balanced (its equilibrium) in these states the ideas of metacentre (M), centre of gravity (G), centre of buoyancy (B) and base or



keel line (K) are used as defined below. These terms enable us to apply mathematics and understand the forces at work in initial stability (to 15° of heel).

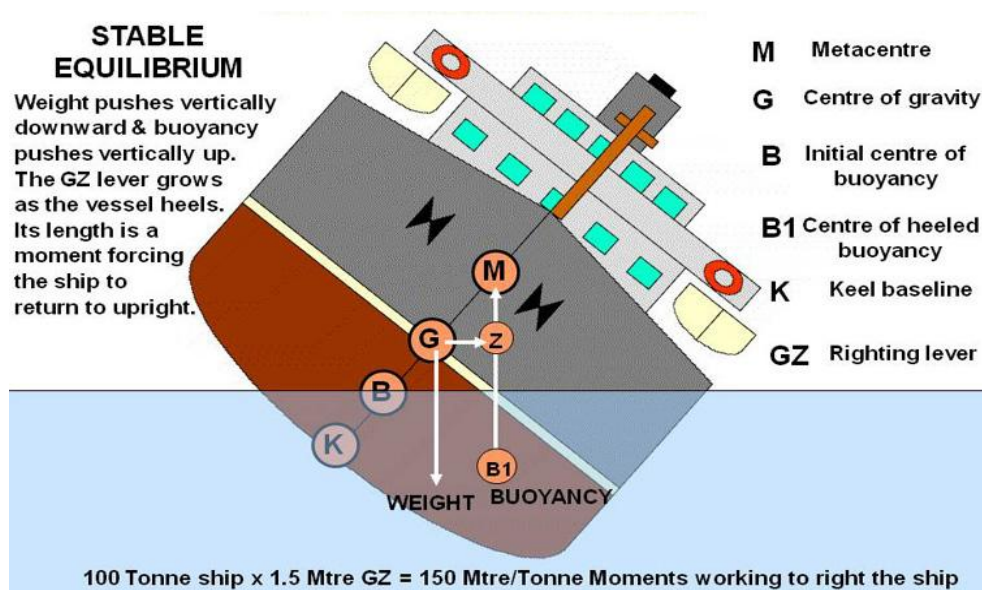
### The metacentre

For initial angles of *heel*, M is thought of as a pivot point with the vessel a pendulum swinging from it. The distance of G below the pivot M is a mark of how *stiff* the vessel is, the greater the GM the faster it will swing back to the upright.



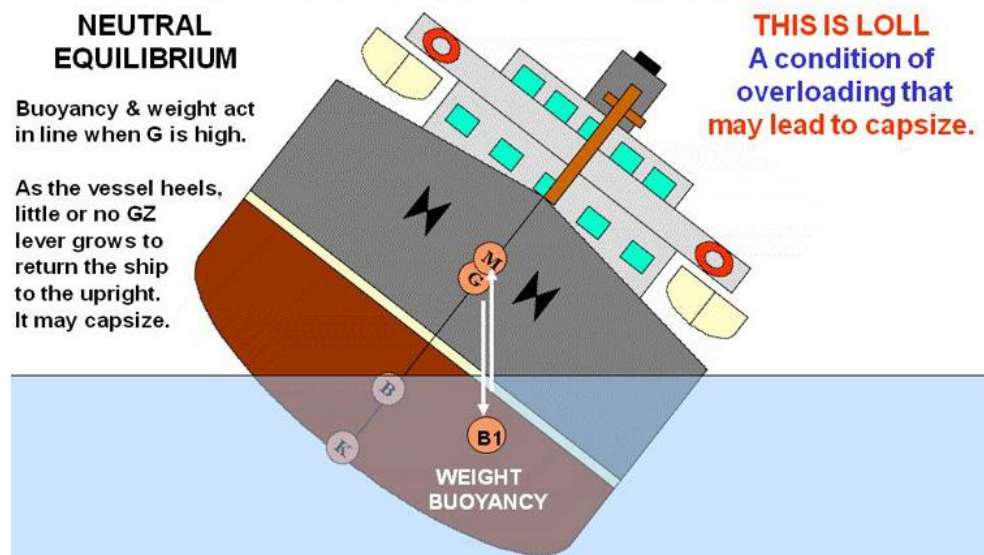
### Stable equilibrium

On heel the centre of buoyancy follows the moving underwater shape being flung out to the position B<sub>1</sub>. For vessels in stable equilibrium, G pushes vertically downward while the offset new B<sub>1</sub> pushes directly upwards. The length of offset is termed the righting lever GZ. Mathematically it is a moment of GZ length times vessel weight, as in the example below, 1.5 metres x 100 tonnes = 150 metre /tonnes - the greater the GZ the greater the moment builds to right the vessel.



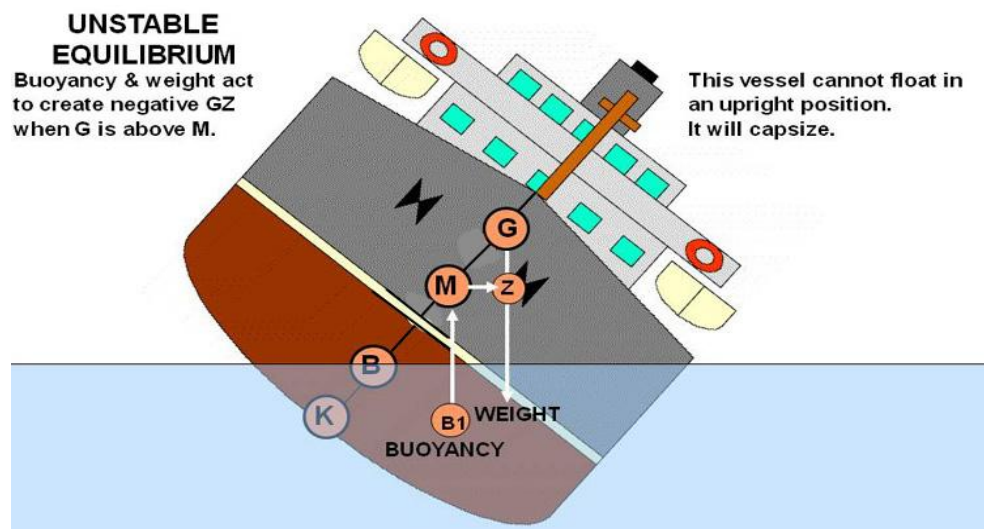
### Neutral equilibrium

A vessel in neutral equilibrium (*loll*) is in a condition of overloading where  $G$  is insufficiently lower than  $M$  to create a  $GZ$  moment and positive righting lever. The vessel may not capsize but will be excessively *tender* or even flop from side to side at the mercy of the last wave that pushed it.

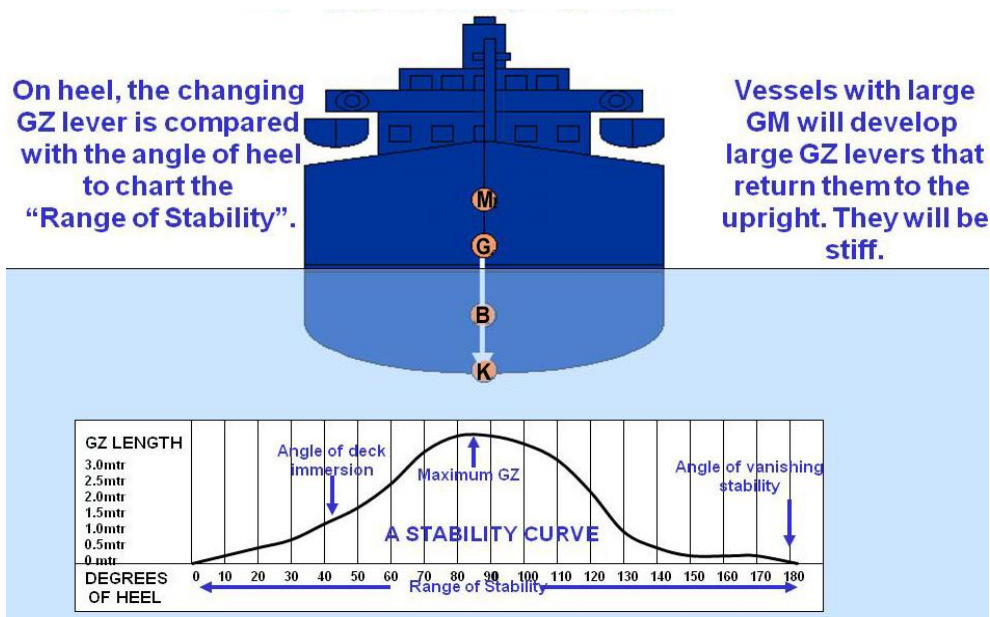


### Unstable equilibrium

A vessel in unstable equilibrium has reverse  $G$  and  $M$  creating a negative  $GZ$  moment forcing it to capsize.



The progressive ability of a vessel to increase its righting lever on heel can be mapped as its *range of stability*. This range is usually only positive until the deck edge goes under, from which time down flooding through weathertight only doors and ports that compromises reserve buoyancy is likely.



### Roll period

A prerequisite of stability is the righting lever created by sufficient GM. The designed GM of a vessel is found for various conditions of loading in its stability book. The status of a vessel's GM is also indicated by the vessel's roll period.

In port, a small vessel (under 100 tonnes) can easily be coaxed to roll by moving weights rhythmically from side to side. The full roll period from upright, through heel to starboard, heel to port and then back to upright can be timed, and by using the formula below a notional GM can be calculated to compare with the stability book. Every vessel is different so a factor has to be included in the formula.

**Change a vessel's roll period (stiffness) indicates change in GM**

$$GM \text{ in mtr} = \left[ \frac{\text{Factor} \times \text{Breadth}}{\text{Time in secs}} \right]^2$$

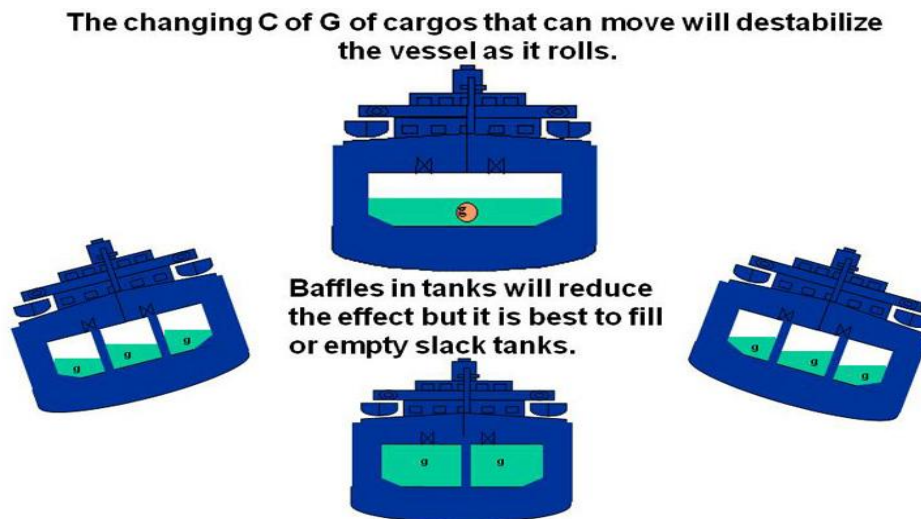
Factors = 0.78 – 0.98  
Dependant on hull form

$$\left[ \frac{0.9 \text{ factor} \times 10 \text{ mtr beam}}{6 \text{ secs roll period}} \right]^2 = [1.5]^2 = GM 2.25 \text{ mtr}$$

While it is unwise to rely on the roll period test for actual GM, a smart master or mate keeps in tune with his vessel's motion, principally noting any change in its roll period.

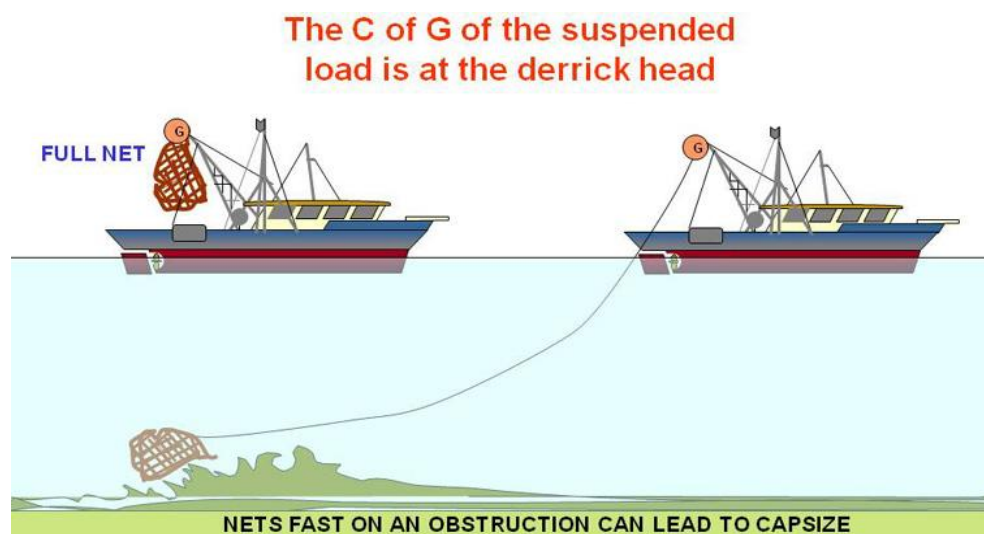
### **Free surface effect**

*Free surface effect* (FSE) is caused by weights moving freely across the vessel during rolling. Typical FSE agents are liquids in part filled tanks, fish, water on deck or trapped in deck cargos (pipes). It takes its name from the exposed surface area of liquids in part filled tanks and can be considered as a virtual rise in G. If the free surface can be reduced by division (by baffles or divided tanks) the surface area is reduced by its square, as below the three tanks reduce the FSE by a factor of nine. Where possible, tanks are best *pressed up* (filled) or emptied.



### **Suspended Weights**

When a weight is hoisted its centre of gravity moves to the point of suspension (the boom head). This will be higher than the vessel's centre of gravity and cause a large rise in G and potential de-stabilisation of the vessel. In the case of a fishing vessel's nets becoming *fast* (stuck on the bottom) the sudden reduction in GM plus the dynamic tug on the vessel from this virtual anchor can cause rapid capsize.



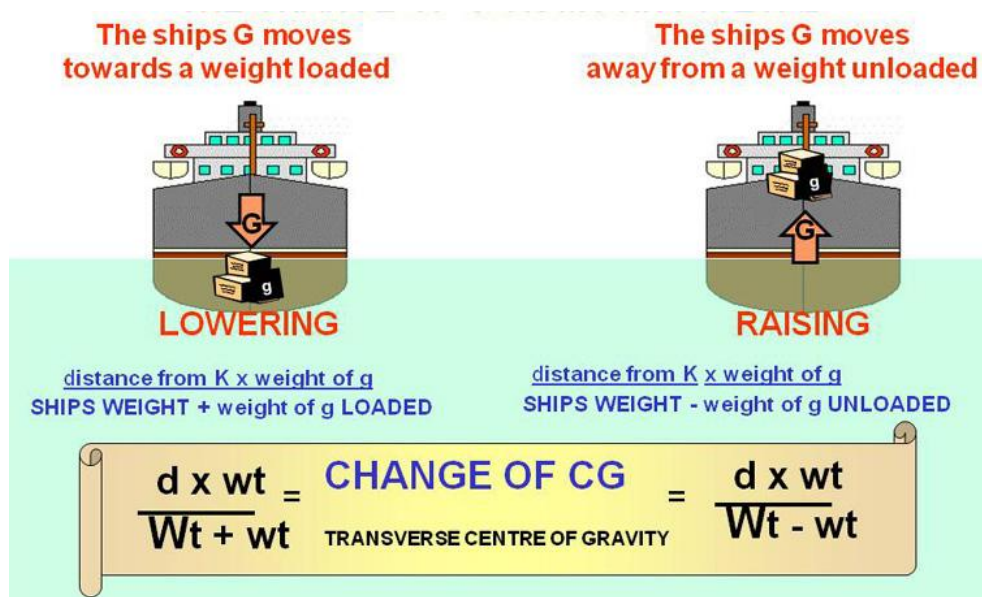


### **Loading and unloading**

When loading a weight (g), the overall centre of gravity (G) of the vessel moves towards its stowed position. When unloading a weight (g), the overall centre of gravity (G) of the vessel moves away from the position where it had been stowed.

Shown below, an additional weight (wt) is loaded onto a vessel so the overall weight or displacement of the vessel (Wt) is increased by  $Wt + wt$ . The balance is changed by the moment ( $d \times wt$ ) so the Vertical Centre of gravity (VCG) is moved towards the position of the weight loaded.

If a weight (wt) is unloaded from a vessel the overall weight of the vessel or displacement (Wt) is decreased by  $Wt - wt$ . The balance is changed by the moment ( $d \times wt$ ) and the VCG is moved away from the position of the weight unloaded.

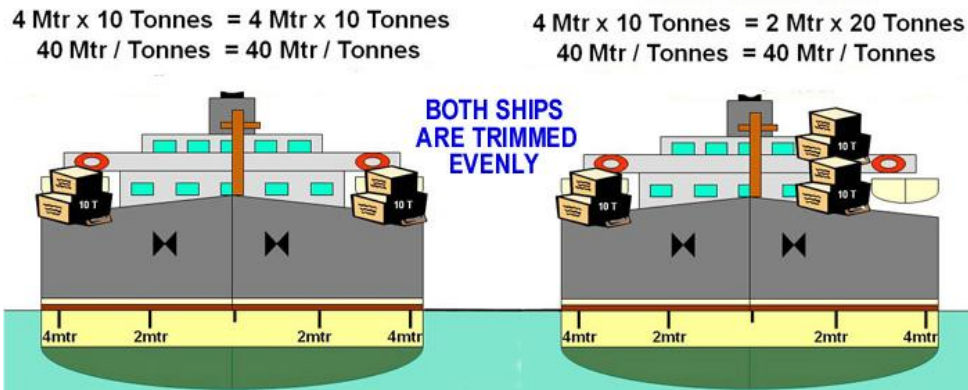


### **List and transverse trim**

An uneven trim or list will occur if weights are not stowed equally on both sides of a vessel as shown below by unequal moments.



However, unequal weights can be loaded with even trim as long as the moments are equal as shown below.



This change in transverse centre of gravity (G) is tabulated as below:

<b>Before loading</b>	<b>Wt</b> tonnes		<b>VCG</b> metres		<b>d x Wt</b> metre/tonnes
Lightship displacement	87.5	x	1.0	=	87.5
<b>After loading</b>	<b>wt</b>		<b>Distance</b>		<b>d x wt</b>
Loaded fwd hold	2.0	x	3.0	=	6.0
Loaded aft hold	4.0	x	1.5	=	6.0
<b>Load displacement</b>	93.5 tonnes				99.5 mtr/tonnes
Loaded VCG	=	$\frac{\text{Load vertical moments}}{\text{Load displacement}}$		=	$\frac{99.5}{93.5} = 1.064 \text{ metres}$

### Summary:

#### *Movement of G (centre of gravity):*

G moves towards loaded weights, away from unloaded & parallel to shifted weights.

A suspended weight acts as though it is located at the point of suspension.

Stability improves if G is lowered and deteriorates if G is lifted.

Free surface effect decreases stability. Its effect is likened to a virtual rise in G.

#### *Stability is improved if:*

Weights already on board are lowered and high weights are removed.

Suspended weights are lowered.

Tanks are kept completely full (pressed up) or completely empty.

#### *Stability is worsened if:*

Weights are removed from low down in the vessel.

Weights already on board are lifted higher.

Weights are lifted on booms.

Tanks have free surfaces.

*Calculating change in G due to loading/unloading:*

A single transverse or longitudinal load change is calculated by comparing the changed moment with the changed displacement, as below.

$$\frac{d \text{ (distance from initial G)} \times wt \text{ (weight loaded/unloaded)}}{Wt \text{ (initial displacement)} \pm wt \text{ (weight loaded/unloaded)}}$$

In practice, a table as below is used to calculate the sums of combined transverse and longitudinal changes due to multiple loaded and/or unloaded items.

<b>Item</b>	<b>Weight Wt + wt</b>	<b>VCG d</b>	<b>V Mom d x wt</b>	<b>LCG d</b>	<b>Long mom d x wt</b>	<b>FSN</b>
Lightship Wt	87.5	1.0	87.5	- 0.295	- 25.81	
Loaded fwd wt	2.0	3.0	6.0	+ 9.95	+ 19.90	1.0
Loaded aft wt	4.0	1.5	6.0	- 20.61	- 82.44	1.5
<b>Loaded</b>	<b>93.5 t</b>		<b>99.5 m/t</b>		<b>- 88.35 m/t</b>	<b>2.5</b>
<b>Loaded</b>	<u>Load vertical moments</u> Load displacement			<u>Load long moments</u> Load displacement		<u>FSN</u> Load d
	= $\frac{99.5}{93.5}$			= $\frac{- 88.35}{93.50}$		= $\frac{2.5}{93.50}$
<b>Movement of G</b>	= 1.064 metres (Transverse G moves upward reducing GM and the vessel's stiffness)			= -0.945 metres (Longitudinal G moves aft causing increased trim by the stern)		= 0.023

Free surface effect (FSE) from partially filled tanks has a substantial destabilising effect. Rather than attempting to calculate the moving liquid's actual G, partially filled tanks are given a free surface numeral (FSN) and regarded as experiencing a virtual rise in transverse G.

In the table above liquid cargos were loaded into the fwd hold (assigned an FSN of 1) and the aft hold (FSN of 1.5). This correction is added to the solid G position and called the fluid G.

Fluid G	= $\frac{1.064 \text{ metres}}{+ 0.023}$ 1.087 metres	Movement of G with virtual rise from FSE	= 0.023 correction
---------	--	---	-----------------------

## Chapter 7: Lifting equipment

### 7.1 Definitions

**Breaking strain (BS)** - is the ultimate load that will break a new rope, wire, shackle or chain and is determined by destructive proof testing.

**Efficiency (E)** - a measure of how well a mechanism converts work input to work output.

$$E \% = \frac{\text{useful work done}}{\text{work supplied}} = \frac{W \times \text{distance } W \text{ moves}}{P \times \text{distance } P \text{ moves}} = \frac{MA}{VR} \times 100$$

**Effort (P)** - the force that is applied to balance or shift a load, measured in kilos (weight) or in Newtons (force).

**Force (F)** - is an energy that can cause a mass to change its velocity, direction and/or deform it. As a force includes magnitude and direction it is a vector quantity. One Newton of force accelerates one kilogram by one metre per second per second.

**Gear ratio (GR)** - a comparison of the number of revolutions that a driving input will revolve the driven output.

**Lever** – an arrangement of load and effort around a fulcrum that changes the effort required to shift a load.

**Lifting machine** - a mechanism that reduces the effort required to lift a load.

**Load (W)** - the weight lifted by a machine or lever.

**Mechanical Advantage (MA)** – A comparison of the load lifted by a mechanism of gears, pulleys or levers to the effort required, expressed by the formula:

$$MA = \frac{\text{load lifted (W)}}{\text{effort applied (P)}}$$

**Moment** - the movement or rotation of a physical quantity about an axis.

**Purchase** - a mechanism that provides a mechanical advantage.



**Safe working load (SWL)** – also called *working load limit* (WLL) refers to a maximum stress for lifting equipment that provides a safety margin that should not be exceeded.

**Stress** - Stress is the internal resistance of a material to an external force exerted over a given area, expressed by the formula:

$$\text{Stress in Newtons/metre}^2 = \frac{\text{Force in Newtons}}{\text{Area in square metres}}$$

**Tackle** - a combination of pulleys and ropes.

**Velocity ratio (VR)** – A comparison of the distance moved by an input mechanism of gears, pulleys or levers to the output mechanism, expressed by the formula:

$$\text{VR} = \frac{\text{distance moved by the effort}}{\text{distance moved by the load}}$$

## 7.2 Strength of rope, wire and chain

Combinations of rope, steel wire rope, chain and their tackles are used in to lift the cargo and equipment loaded and unloaded on vessels. Shackles, hooks and strops allow for secure hitching and unhitching. But to do this safely and reliably, all parts of the lifting system must be strong enough to do the job.

*Breaking strain* (BS) is the ultimate load that will break a new rope, wire, shackle or chain and can be determined by destructive proof testing. *Safe working load* (SWL) or *working load limit* (WLL) refers to a stress that provides a safety margin. This safety margin is essential to allow for equipment age, quality consistency and operating variables. In shore side industry a safety margin of SWL 1/5<sup>th</sup> the BS applies, but in the demanding marine environment SWL 1/6<sup>th</sup> of the BS is a typical precaution. Due to the advanced technologies found in modern materials the most accurate determinations of breaking BS and SWL are found in the manufacturer's specifications. However any seaman that is unable to make basic evaluations of safety margins for lifting equipment is at risk of harming self and others.

### **Rope and wire**

The ultimate strength of laid ropes depends much upon the quality of fibre and the process of manufacture. A rope's strength is greatly increased by the lengths of the constituent fibres. Relative order of rope fibre strength is:

Short natural fibres of - coir (coconut husk), sisal, manila and hemp and

Long synthetic fibres of - polythene, polypropylene, terylene, nylon and kevlar.

The diameter of rope is measured in millimetres (mm). The marine safety factor is 1/6. Thus SWL can be taken to be 1/6 of the BS. Splicing a rope reduces its strength by at least 10%. Knots in a rope reduce its breaking strength (BS) by up to 50%.

As different rope fibres are of differing strength, a rough approximation by calculation must include a strength factor (ranging from 1 to 8) within the formula:

$$\text{SWL of a Rope type} = \text{Diameter}^2 (\text{mm}) \times \text{Factor (kgs)}$$

(where D is diameter of rope in millimetres and F is a factor of safety).

<b>Material</b>	<b>Factor</b>	<b>Approx. S.W.L.</b>
Natural Fibre Rope	1	D <sup>2</sup>
Silver rope (flat spin taniklon) and Polyethylene (Staple)	1.2	1.2D <sup>2</sup>
Polypropylene and Polyethylene (Mono)	1.8	1.8D <sup>2</sup>
Polyester (Terylene)	2.0	2.0D <sup>2</sup>
Polyamide (Nylon) > 50 mm	2.5	2.5D <sup>2</sup>
Polyamide (Nylon) < 50 mm	3.0	3.0D <sup>2</sup>
Spectra (Kevlar)	6.0	6.0D <sup>2</sup>
Steel Wire Rope	8.0	8.0D <sup>2</sup>

#### **Example 1:**

The SWL of 12 mm Polypropylene SWL (kgs) = 1.8D<sup>2</sup>

$$= 1.8 \times 144$$

$$= 259.2 \text{ kgs}$$

The BS of 12 mm Polypropylene = SWL x 6

$$= 1555.2 \text{ kgs}$$

*Example 2:*

The SWL of 12 mm Wire Rope (6 x 24) =  $8D^2$

$$= 8 \times 144$$

$$= 1152 \text{ kgs}$$

The BS of 12 mm Wire Rope (6 x 24) =  $SWL \times 6$

$$= 6912 \text{ kgs}$$

The examples above estimate the BS of same diameter poly rope to be about 1½ tonnes compared to steel wire rope at about 7 tonnes. While this illustrates the strength of steel wire rope it also warns of the severity of personnel injury resulting from the whiplash recoil of steel wire ropes if stressed past their breaking point.

Although common practice for fibre and wire rope in marine work allows a safety factor of 6, wire slings encountered during unloading may be rated to the lesser shore based industry safety factor of 5.

### **Chain**

As previously stated, due to the advanced technologies found in modern materials accurate determinations of breaking BS and SWL are best found in the manufacturer's specifications. Oblong (round) link chain is commonly used for lifting operations and stud link chain for anchor work. With multiple legged chain slings, the number, configuration and angle of stress on component chains are critical determinants in any strength assessment. For further information about *dogging practice* (use of lifting equipment) refer to the WorkCover guidelines.

Chain found on vessels may well be of foreign origin and be sized with imperial measures. The conversion will approximate as:

$$\begin{array}{llll} 1/4 \text{ inch} = 6\text{mm} & 5/16 \text{ inch} = 8\text{mm} & 3/8 \text{ inch} = 10\text{mm} & 1/2 \text{ inch} = 12\text{mm} \\ 3/4 \text{ inch} = 18\text{mm} & 7/8 \text{ inch} = 22\text{mm} & 1 \text{ inch} = 25\text{mm} & \end{array}$$

Under ISO standard classification, Grade 3 round/oblong link chain is of mild steel while Grades 4 to 8 are progressively hardened and tempered tensile steels. Under traditional marine anchor large stud link chain classification, Grade 1 stud link chain is of mild steel while Grades 2 to 3 are progressively hardened steels. When calculating SWL or BS of chain, beware of the different identification systems and if in doubt, assume that your chain is of the lowest grade.

Although common practice for chain in marine work allows a safety factor of 5, chain encountered during unloading may be rated to the lesser shore based industry safety factor of 4. Estimates shown for SWL for smaller size round link chain are based on the formula and table below:

***3 x Diameter<sup>2</sup> x Grade of chain***

<b><i>Round/Oblong Link Chain &lt;12.5 mm</i></b>					
<b><i>Grade</i></b>		<b><i>Type</i></b>	<b><i>Stamp alternatives</i></b>	<b><i>SWL kgs</i></b>	<b><i>or SWL kgs</i></b>
Grade 3	L	Mild steel	3, 30 or L	$3D^2 \times 3$	$0.3D^2 \times 30$
Grade 4	M	High tensile steel	04, 4, 40 or M	$3D^2 \times 4$	$0.3D^2 \times 40$
Grade 5	P	High tensile steel	50 or P	$3D^2 \times 5$	$0.3D^2 \times 50$
Grade 6-7	S	Transport only (not for lifting)	60, 70, 75 or S	$3D^2 \times 6/7$	$0.3D^2 \times 60$
Grade 8	T	Alloy steel (lifting standard)	8, 80, or 800, HA PBB, CM or T	$4D^2 \times 8$	$0.4D^2 \times 80$

***Example 1:***

The SWL of 9 mm ISO Grade 3 chain (**L**) =  $3D^2 \times \text{Grade}$

$$= 3 \times 81 \times 3$$

$$= 729 \text{ kgs}$$

The BS of 9 mm ISO Grade 3 chain (**L**) =  $\text{SWL} \times 5$

$$= 729 \times 5$$

$$= 3645 \text{ kgs}$$

***Example 2:***

The SWL of 12 mm ISO Grade 8 chain (**T**) =  $4D^2 \times \text{Grade}$

$$= 4 \times 144 \times 8$$

$$= 4608 \text{ kgs}$$

The BS of 12 mm ISO Grade 8 chain (**T**) =  $\text{SWL} \times 5$

$$= 4608 \times 5$$

$$= 23040 \text{ kgs}$$

It remains practice for larger stud link anchor chain to use the classification and formula:

<b>Stud Link Anchor Chain &gt;12.5 mm to &lt;120 mm</b>	
<b>Grade and type</b>	<b>BS tonnes</b>
Grade 1 Mild Steel	$\frac{(20D^2)}{600}$
Grade 2 Special Quality Steel	$\frac{(30D^2)}{600}$
Grade 3 Extra Special Quality Steel	$\frac{(43D^2)}{600}$

*Example 1:*

$$\begin{aligned}
 \text{The BS of 16 mm stud link anchor Grade 1 chain} &= 20D^2 \div 600 \\
 &= (20 \times 16 \times 16) \div 600 \\
 &= 8.53 \text{ tonnes}
 \end{aligned}$$

$$\begin{aligned}
 \text{The SWL of 16 mm stud link anchor Grade 1 chain} &= BS \div 5 \\
 &= 1.7 \text{ tonnes}
 \end{aligned}$$

*Example 2:*

$$\begin{aligned}
 \text{The BS of 20 mm stud link anchor Grade 2 chain} &= 30D^2 \div 600 \\
 &= (30 \times 20 \times 20) \div 600 \\
 &= 20 \text{ tonnes}
 \end{aligned}$$

$$\begin{aligned}
 \text{The SWL of 20 mm stud link anchor Grade 2 chain} &= BS \div 5 \\
 &= 4 \text{ tonnes}
 \end{aligned}$$

### **Check your progress with strength of rope, wire and chain:**

*Question 1:*

What are the SWL and BS of 12 mm nylon rope?

*Question 2:*

What are the SWL and BS of 10 mm steel wire rope?

*Question 3:*

What are the SWL and BS of 16 mm polypropylene rope?

*Question 4:*

What are the SWL and BS of 12 mm hemp rope that has a knot in it?

*Question 5:*

What are the SWL and BS of 10 mm ISO Grade 3 chain marked with stamp **(L)**?

*Question 6:*

What are the SWL and BS of 6 mm ISO Grade 8 chain marked with stamp **(80)**?

*Question 7:*

What is the BS of 25 mm stud link anchor Grade 1 chain?

*Question 8:*

What is the BS of 2 inch stud link anchor Grade 3 chain?

***Answers to strength of rope, wire and chain:***

*Answer 1:*

The SWL of 12 mm Nylon SWL (kgs)       $= 3D^2 = 3 \times 144 = 432$  kgs  
The BS of 12 mm Nylon                       $= 432 \times 6 = 2592$  kgs

*Answer 2:*

The SWL of 10 mm SWR SWL (kgs)       $= 8D^2 = 8 \times 100 = 800$  kgs  
The BS of 10 mm SWR                       $= 800 \times 6 = 4800$  kgs

*Answer 3:*

The SWL of 16 mm Polyprop SWL (kgs)  $= 1.8D^2 = 1.8 \times 256 = 460.8$  kgs  
The BS of 16 mm Polyprop                 $= 460.8 \times 6 = 2764.8$  kgs

*Answer 4:*

The SWL of 12 mm Hemp SWL (kgs)       $= D^2 = 144 = \times 50\% = 72$  kgs  
The BS of 12 mm Hemp                       $= 144 \times 6 = 864 \times 50\% = 432$  kgs

*Answer 5:*

The SWL of 10 mm ISO Grade 3 chain **(L)**  $= 3D^2 \times \text{Grade} = 3 \times 100 \times 3 = 900$  kgs  
The BS of 10 mm ISO Grade 3 chain **(L)**  $= \text{SWL} \times 5 = 900 \times 5 = 4500$  kgs

*Answer 6:*

The SWL of 6 mm ISO Grade 8 chain **(80)**  $= 4D^2 \times \text{Grade 8 or } 0.4D^2 \times 80 = 1152$  kgs  
The BS of 6 mm ISO Grade 8 chain **(80)**  $= \text{SWL} \times 5 = 1152 \times 5 = 5760$  kgs

*Answer 7:*

The BS of 25 mm stud link anchor Grade 1 chain  $= 20D^2 \div 600 = 20.83$  tonnes

*Answer 8:*

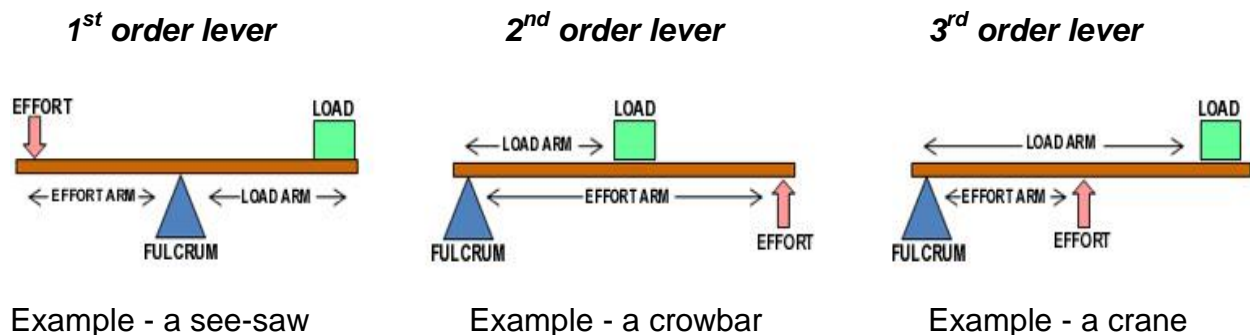
The BS of 2 inch (50 mm) stud link Grade 3 chain  $= 43D^2 \div 600 = 179.17$  tonnes

## 7.3 Levers and moments

### ***The orders of simple levers***

Simple levers are classified by the relative positions of the load, effort and fulcrum (pivot point). Differing examples include see-saws, crow bars and mobile cranes. Each configuration enables shifting a load as is most convenient to a particular situation. The effort required to move, lift or balance the load around the fulcrum is determined by the configuration and can be calculated by using moments.

### **Simple levers**



*Moments* are the movement or rotation of a quantity around an axis and are expressed as:

$$\text{Moment} = \text{Mass} \times \text{Its distance from the fulcrum}$$

Effort arm and Load arm are the respective distances of the effort and the load from the fulcrum. When there is equilibrium (balance), the moments around the fulcrum will equal zero and are expressed as:

$$\text{Effort} \times \text{Effort arm} = \text{Load} \times \text{Load arm}$$

or

$$\text{Effort} = \frac{\text{Load} \times \text{Load arm}}{\text{Effort arm}}$$

In the drawing above, if our load was 2 tonnes and the length of our beam was 4 metres then the effort required to balance the levers will be:

$$\text{1<sup>st</sup> order lever} - \text{Effort in tonnes} = \frac{2 \text{ tonne} \times 2 \text{ mtrs}}{2 \text{ mtrs}} = 2 \text{ tonnes}$$

$$\text{2<sup>nd</sup> order lever} - \text{Effort in tonnes} = \frac{2 \text{ tonne} \times 2 \text{ mtrs}}{4 \text{ mtrs}} = 1 \text{ tonne}$$

$$\text{3<sup>rd</sup> order lever} - \text{Effort in tonnes} = \frac{2 \text{ tonne} \times 4 \text{ mtrs}}{2 \text{ mtrs}} = 4 \text{ tonnes}$$

Our calculations reveal that in using different orders of levers (relative positions of fulcrum and load) the required effort can be traded against the ultimate distance that the load is moved. The comparison of the distance moved by an input to an output is called the *velocity ratio* (VR), expressed by the formula:

$$\text{Velocity Ratio} = \frac{\text{distance moved by the effort}}{\text{distance moved by the load}}$$

In short, application of the appropriate lever or mechanism can enable a weight (load) to be moved over a given distance either slowly with small effort or quickly with a large effort. Moving large weights, albeit a short distance, with small effort is called the *mechanical advantage* of a lever or lifting system. *Mechanical Advantage* (MA) is a comparison of the load lifted by a mechanism of gears, pulleys or levers to the effort required, expressed by the formula:

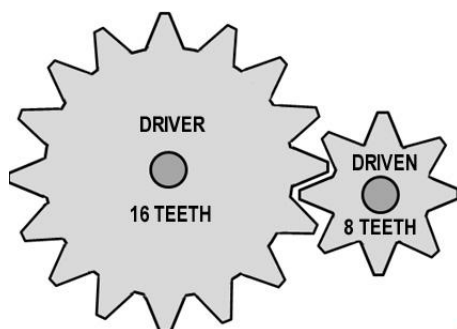
$$\text{Mechanical Advantage} = \frac{\text{load lifted (W)}}{\text{effort applied (P)}}$$

The *Velocity ratio* and the *Mechanical advantage* of a lifting system are closely related. If it were not for the energy loss in all systems caused by friction and heat their ratios would be the same. Efficiency (E) is a measure by percentage of how well a mechanism converts work input to work output, expressed as:

$$E \% = \frac{\text{useful work done}}{\text{work supplied}} = \frac{\text{Load W x distance W moves}}{\text{Effort P x distance P moves}} = \frac{MA}{VR} \times 100$$

### **Simple rotary machines**

Simple machines manipulate difference in velocity ratio to gain mechanical advantage by utilising pulleys, gears and blocks & tackles. Shown below, the larger driver gear cog has 16 teeth and the smaller driven gear cog 8 teeth creating a VR of 2 to 1, which gives MA of 2 to 1 (assuming a 100% efficient system).





Similarly, the relative circumferences of trains of pulleys will change the velocity ratio of the system.



*Example 1:*

In the automobiles gear train above, the driven 1<sup>st</sup> gear cog (48 teeth) meshes with the layshaft driver cog (12 teeth). What is the gear ratio and its velocity ratio?

*Calculation 1:*

$$GR = \frac{\text{Teeth on driver cog}}{\text{Teeth on driven cog}} = \frac{12}{48} = \frac{1}{4} \quad \text{The driver cog turns 4 times of to turn the driven once}$$

$$VR = \frac{\text{distance moved by the effort (driver)}}{\text{distance moved by the load (driven)}} = \frac{4}{1} = 4 \text{ to } 1$$

*Example 2:*

In the automobiles gear train above, the driven 5<sup>th</sup> gear cog (9 teeth) meshes with the layshaft driver cog (27 teeth). What is the gear ratio and its velocity ratio?

*Calculation 2:*

$$GR = \frac{\text{Teeth on driver cog}}{\text{Teeth on driven cog}} = \frac{27}{9} = \frac{3}{1} \quad \text{The driver cog turns once of to turn the driven three times}$$

$$VR = \frac{\text{distance moved by the effort (driver)}}{\text{distance moved by the load (driven)}} = \frac{1}{3} = 1 \text{ to } 3$$

Assuming 100% efficient systems, the following examples show the relationship of MA, Load and Effort.

*Example 3:*

Determine the required MA of a lever system where an effort of 600N (Newtons) is available to lift a load of 2.40 kN (kilo Newtons).

*Calculation 3:*

$$MA = \frac{\text{Load}}{\text{Effort}} = \frac{W}{P} = \frac{2400}{600} = 4 \text{ to } 1$$

*Example 4:*

Alternatively, if another lever system with an effort of 300N (Newtons) lifted a load of 2.40 kN (kilo Newtons), its MA can be calculated as:

*Calculation 4:*

$$MA = \frac{\text{Load}}{\text{Effort}} = \frac{W}{P} = \frac{2400}{300} = 8 \text{ to } 1$$

The Second system's mechanical advantage is greater - this system is more advantageous than example 1.

*Example 5:*

Determine the effort required to lift a load of 1.6kN given the mechanical advantage of a lifting system is 4 to 1.

*Calculation 5:*

$$\text{Effort} = \frac{\text{Load}}{MA} = \frac{W}{MA} = \frac{1600}{4} = 400\text{N}$$

*Example 6:*

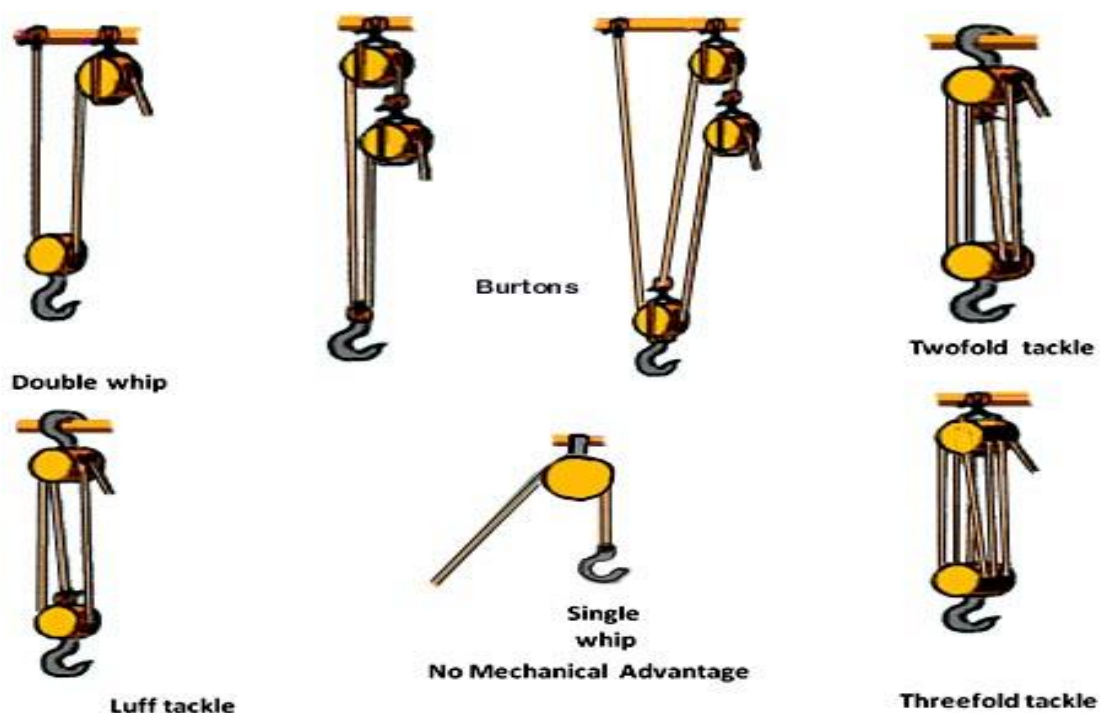
Determine the effort required to lift a load of 3kN given the mechanical advantage of a lifting system is 6 to 1.

*Calculation 6:*

$$\text{Effort} = \frac{\text{Load}}{MA} = \frac{W}{MA} = \frac{3000}{6} = 500\text{N}$$

## 7.4 Blocks, tackles and purchases

The *single whip* shown below is convenient for lifting loads as the weight of a person hauling can offset the weight of the load lifted, but there is no mechanical advantage at work in this rig. *Mechanical advantage* (MA) is gained when a smaller effort is harnessed to move a greater load at a slower speed. Levers, gears and purchases (blocks and tackles) are all examples of these mechanisms that trade ultimate output load capability for input effort over a longer time.

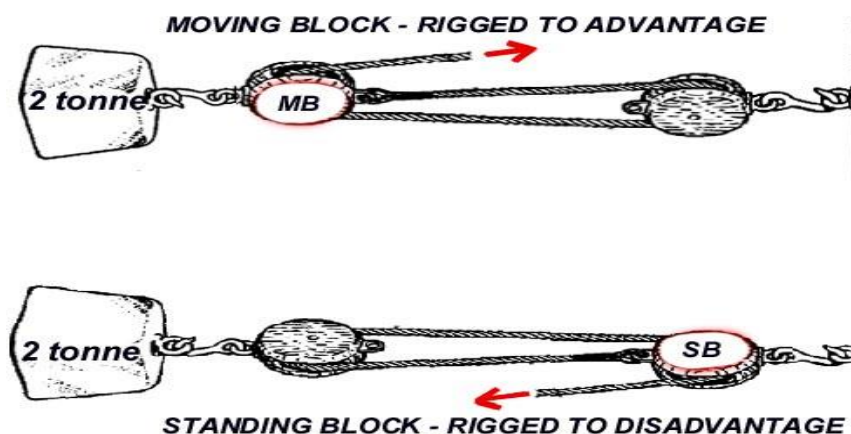


Mechanical advantage is achieved with blocks and tackles by using ropes threaded through pulley sheaves, the effect of which is to lengthen the rope hauled (the effort required) compared to the shorter closing distance between the blocks (the load lift). Blocks and tackles can be configured with multiple pulley sheaves and/or block arrangements that alter the velocity ratio and ultimate mechanical advantage ratio.

The following description of blocks and tackles does not allow for energy losses due to inefficiencies (i.e. for simplicity VR is assumed to be equal to MA). An ideal purchase for a lift can be configured that lifts or drags a load at a determined rate and theoretical effort by the using additional pulley sheaves.

Where the tackle is rigged to *disadvantage* the mechanical advantage ratio is the same as the number of sheaves in the tackle. In this configuration the rope is hauled from where it exits the stationary block of sheaves, not from the moving block with its attached load. Shown above, *Luff tackle* has three sheaves and a MA of 3 to 1. Without considering reductions due to friction, this tackle will lift a load) three tonnes with a pull (an effort) of one tonne. The *Twofold tackle* has four sheaves and a MA of 4 to 1. This tackle will require an effort of one tonne to shift four tonnes. Simply put, if the effort is hauled from the standing block then the tackle is rigged to *disadvantage* and the *mechanical advantage ratio* is equivalent to the number of sheaves in the system (or ropes that join the blocks). For example, *Threefold tackle* has six sheaves, six ropes joining the blocks, a MA of 6 to 1 so an effort of one tonne will shift a load of six tonnes.

Below it can be seen that a simple tackle can be rigged in two ways. The effort could haul from the standing block & load (static) or from the moving block & load. While they look similar, more rope has to be pulled to close the gap between the blocks (to shift the load) when pulled from the moving block.



When rigged to *advantage* (hauled from the moving block) of sheaves the *mechanical advantage ratio* is calculated as the number of sheaves plus one. For example, *Gun tackle* shown above rigged to advantage has two sheaves, three ropes joining the blocks and a MA of 3 to 1, so an effort of one tonne will shift a load of three tonnes.

More complicated rigs, where one tackle's haul rope leads to another tackle, are called *burtons*. The same mechanical advantage ratios apply for the constituent tackles, but the mechanical advantage of the whole is calculated as the product of the constituent tackles. For example, a twofold tackle (4 sheaves) rigged in combination with a gun tackle (two sheaves), both rigged to disadvantage, will have a MA of  $4 \times 2 = \text{MA } 8:1$ .

**Check your progress with mechanical advantage:** (assume  $VR = MA$ )

*Question 1:*

What is the VR/MA of a gun tackle rigged to disadvantage?



*Answer 1:*  
2:1

*Question 2:*

What effort is required to shift a two tonnes with a gun tackle rigged to disadvantage?

*Answer 2:*  
1 tonne

*Question 3:*

What is the VR/MA of a gun tackle rigged to advantage?



*Answer 3:*  
3:1

*Question 4:*

What effort is required to shift a 300kg with a gun tackle rigged to advantage?

*Answer 4:*  
100 kg

*Question 5:*

What is the VR/MA of a twofold tackle rigged to disadvantage?



*Answer 5:*  
4:1

*Question 6:*

What effort is required to shift a 100 kg with a twofold tackle rigged to disadvantage?

*Answer 6:*  
25 kg

*Question 7:*

What is the VR/MA of a twofold tackle rigged to advantage?



*Answer 7:*  
5:1

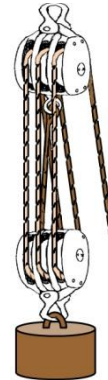
*Question 8:*

What effort is required to shift a 50 kg with a twofold tackle rigged to advantage?

*Answer 8:*  
10 kg

*Question 9:*

What is the VR/MA of a threefold tackle rigged to disadvantage?



*Answer 9:*  
6:1

*Question 10:*

What effort is required to shift a 300kg with a threefold tackle rigged to disadvantage?

*Answer 10:*  
50 kg

*Question 11:*

What is the VR/MA of a threefold tackle rigged to advantage?



*Answer 11:*  
7:1

*Question 12:*

What effort is required to shift a 350 kg with a threefold tackle rigged to advantage?

*Answer 12:*  
50 kg

## 7.5 Friction and efficiency

### **Friction**

In a completely efficient machine MA would be equal to VR. However, all machines are subject to inefficiencies largely caused by friction and consequent energy loss as heat. If friction or other losses were discounted, then the effort required to lift the load could be called the Ideal Effort.

$$\text{Ideal Effort} = \frac{\text{Load (W)}}{\text{Velocity ratio (VR)}}$$

*Example 1:*

If a load is 250N and the velocity ratio is 2.5, what is the ideal effort?

*Calculation 1:*

$$\text{Ideal Effort} = \frac{W}{VR} = \frac{250}{2.5} = 100N$$

The effort specifically required to overcome friction is equal to the actual effort applied less the ideal effort, calculated by applying the following formula:

$$\text{Effort to overcome friction} = \text{Actual Effort} - \text{Ideal Effort} = P - \frac{W}{VR}$$

*Example 2:*

If a load is 250N, the velocity ratio is 2.5, and the actual effort is ?

$$\text{Effort to overcome friction} = P - \frac{W}{VR} = 110 - \frac{250}{2.5} = 10N$$

### **Efficiency**

In comparing a machine's MA and VR its efficiency can be calculated. Efficiency is generally expressed as a percentage and is calculated using the following formula:

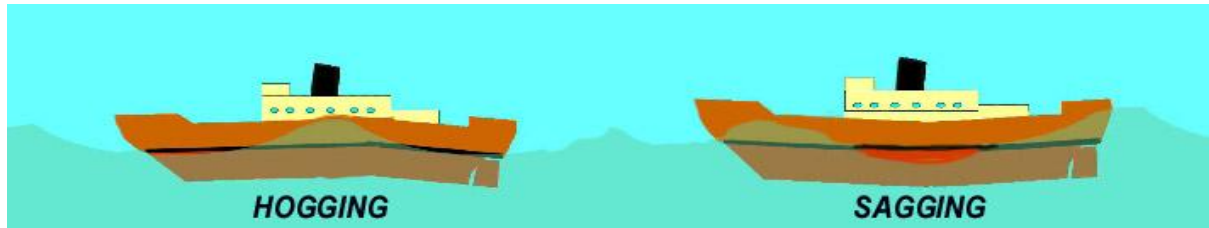
$$E \% = \frac{\text{useful work done}}{\text{work supplied}} = \frac{W \times \text{distance } W \text{ moves}}{P \times \text{distance } P \text{ moves}} = \frac{MA}{VR} \times 100$$

## Chapter 8: Structure and strength

### 8.1 Stress and strain

A vessel hull is subject to forces of weather and waves tending to distort the hull and structure.

The vessel's longitudinal structures of keel and plank are flexed as it is alternatively supported centrally (hogging) or at its extremities (sagging).



Pitching stresses the forefoot and can create a bellows effect in the bow called *panting*.



Rolling tends to deform the squareness of the transverse structure, called *racking*.



Point loads act on its structure and machinery during operations, as below:





### **Force**

Force is the quantity that changes the motion of a body measured in Newtons. One Newton accelerates one kilogram by one metre per second per second.

### **Stress**

Stress is the internal resistance of a material to an external force exerted over a given area, expressed by the formula:

$$\text{Stress in Newtons/metre}^2 = \frac{\text{Force in Newtons}}{\text{Area in square metres}}$$

### **Strain**

An object under stress may change shape - such change is known as strain, in Standard International units measured as metres per metre.

Forces and stress can be described by their effect in deforming a material, for instance by compression, tension, shear, torsion and flex:

#### **Compression**

If the force has a tendency to squash the object, the stress is called compressive stress.

#### **Tension**

If the force has a tendency to stretch the object, the stress is called a tensile stress.

#### **Shear**

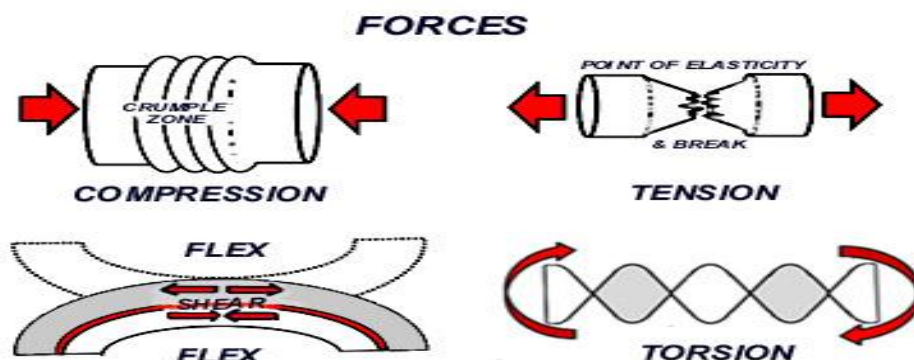
If the force has a tendency to rip the material apart, the stress is called shear stress.

#### **Torsion**

If the force has a tendency to twist the material (e.g. a rotary component such as between a coupling and a transmission shaft), the stress is called torsion.

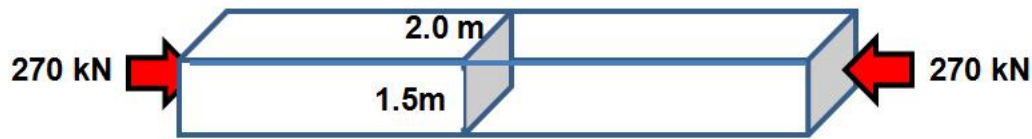
#### **Flex**

If the force has a tendency to bend the material repeatedly (e.g. setting up alternating compression and tension stresses), the stress is called flex. Repeated bending causes friction and heat leading to structural collapse.



\* Note -With compressive/tensile forces, the cross sectional plane of the material, which is at right angles with the force, is the area carrying the force.

Example 1:



Shown above, a compressive force of 270kN (kiloNewtons) is applied to a solid timber beam of cross section 2 metres by 1.5 metres. Calculate the cross sectional area resisting the load. What stress is experienced?

Calculation 1:

$$\text{Area} = 2\text{m} \times 1.5\text{m} = 3\text{m}^2$$

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{270 \times 1000}{3} = \frac{270000\text{N}}{3} = 90000 \text{ N/m}^2$$

### Deformation

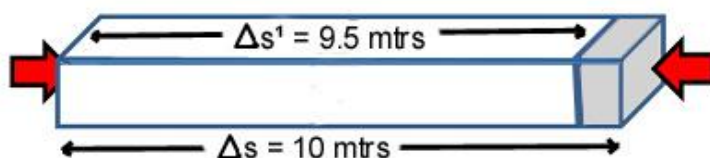
When an object is changed in size or shape by a force it is deformed from its original state. In engineering calculations this deformation is known as strain and denoted by the symbol  $\epsilon$  (epsilon). Below, a chosen line of dimension is indicated by  $\Delta$ .

**Normal or Longitudinal Strain** describes when the force shortens (compresses) or lengthens (tensions) the object parallel to the chosen line of the shape (i.e. length or width). The change of length per unit of length is measured by dividing the change in length by the original length.

$$\epsilon(\text{strain}) = \frac{\Delta s' (\text{changed length}) - \Delta s (\text{original length})}{\Delta s (\text{original length})}$$

Example 2:

What is the normal strain resulting from compressive stress on a post squashed from 10 metres in length to 9.5 metres in length?



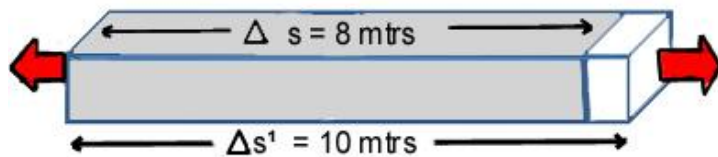
Calculation 2:

$$\text{Normal strain} = \frac{9.5 - 10}{10} = - \frac{0.5}{10} = - \frac{1}{20} = -5\%$$

$\epsilon(\text{strain})$  = compressed by 0.05 mtrs/mtr or -5%.

Example 3:

What is the normal strain resulting from tensile stress on a hook stretched from 8 centimetres in length to 10 centimetres in length?



Calculation 3:

$$\text{Normal strain} = \frac{10 - 8}{8} = + \frac{2}{8} = + \frac{1}{4} = +25\%$$

$\epsilon(\text{strain})$  = stretched by 0.25 mtrs/mtr or +25%.

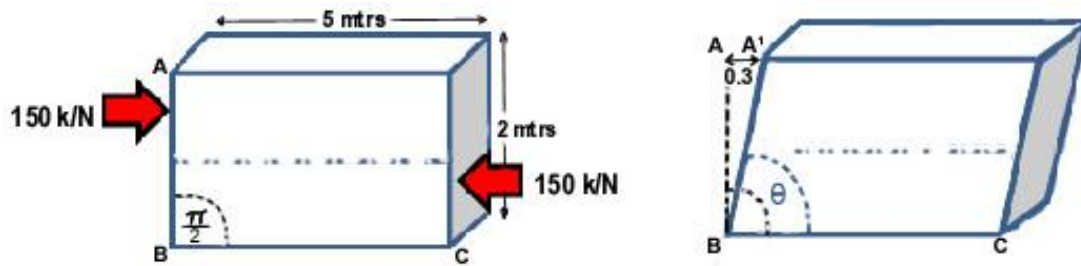
You will notice in the examples above that negative strain is a shortening due to compression whereas positive strain is a lengthening due to tension.

**Shear strain** is the *perpendicular* deformation from the original line of the shape (rather than *parallel* as in normal strain). *True shear strain* is defined as the change in the angle (in radians) from the initial line. *Engineering shear strain* is defined as the tangent of that angle, and is equal to the length of deformation divided by the perpendicular length in the plane along which the force is applied. See more about radians in Chapter 2.5 Trigonometry.

$$y(\text{shear strain}) = \text{initial angle (in radians)} - \text{deformed angle (in radians)}$$

Example 4:

The box below has deformed due to force applied above and below its centreline. What is the shear strain experienced?



Calculation 4:

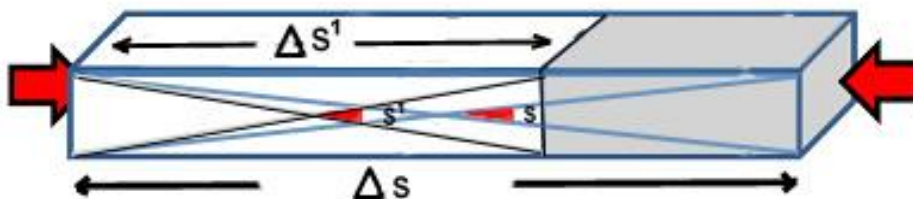
$$y \text{ (shear strain)} = \text{Angle } ABC \text{ (in radians)} - \text{Angle } A'BC \text{ (in radians)}$$

$$y \text{ (shear strain)} = \frac{\pi}{2} \text{ (in radians)} - 81.5^\circ \text{ (in radians)}$$

$$y = 1.57 \text{ radians} - 1.42 \text{ radians} = 0.15 \text{ radians}$$

\*Note -  $360^\circ = 2\pi$  radians,  $180^\circ = \pi$  radians,  $90^\circ = \pi/2$  radians  
- if the deformed angle is parallel to the line, the shear strain will be infinite.

In the previous examples it has been assumed that strain has occurred in a linear manner - that the stretch or squash has been of an even rate over the length of the deformed section of material. In reality the strain's extent may vary over the line's length. As a consequence, strain is most accurately calculated over the shortest of sections, if not at a points along a line. Additionally, strain is usually a mixture of normal and shear strains.



As shown below, on the face of it normal strain has occurred in compressing the dimension of the beam from  $\Delta s$  to  $\Delta s'$ . However, for this to happen the material has also been internally deformed at the angle  $s$  to angle  $s'$  - this angular deformation is shear strain.

## 8.2 Strength

### **Ultimate Tensile Strength - UTS**

The strength of a material is measured as the stress that is required to break it.

*Ultimate Tensile Strength* (UTS) expresses the strength under tension calculated as:

$$\text{UTS} = \frac{\text{Maximum breaking force}}{\text{Cross sectional area}}$$

*Example 1:*

A bar of steel 50mm wide and 25mm high was stretched in a tensile testing machine. The bar fractured at a pull of 100kN. What was its Ultimate Tensile Strength?

*Calculation 1:*

$$\text{UTS} = \frac{100,000\text{N}}{50 \times 25}$$

$$\text{UTS} = 80 \text{ N/mm}^2$$

### **Safety factors**

The breaking force of a material is benchmarked by such destructive testing of newly manufactured samples. That degree of strength is then attributed to all subsequent samples constructed by the same process with the same quality of materials. But there may be variations in the raw materials or manufacturing conditions, the material may corrode with age or deform with prolonged overloading or snatch loads.

Engineers allow for such potential variation from the tested breaking force by two different strategies. The first strategy is a scheduled service life with regular component (parts) replacement. This is common in high performance machinery, as in the aircraft industry, where low weight and fine tolerances are a necessity. The second strategy is to restrict the usage of the structure to situations where stress is lower than the maximum anticipated allowing for future deterioration. This latter approach is common in the marine industry where weight is not critical. The usage limit is called the *safe working load* or *working load limit*, and is calculated as:

$$\text{Factor of safety (FOS)} = \frac{\text{Breaking stress}}{\text{Working stress}}$$

Working Stress is calculated as:

$$\text{Working stress} = \frac{\text{Breaking stress}}{\text{Factor of safety}}$$

**Example 1:**

What is the Factor of safety (FOS) of a mast assuming a UTS of 400 MN/m<sup>2</sup> and working stress of 25 MN/m<sup>2</sup>?

**Calculation 1:**

$$\text{Factor of safety (FOS)} = \frac{\text{Breaking stress}}{\text{Working stress}}$$

$$\text{Factor of Safety} = \frac{400}{25} = 16$$

**Example 2:**

What is the working stress if a piston rod has a FOS of 16 and UTS of 480 MN/m<sup>2</sup>?

**Calculation 2:**

$$\text{Working stress} = \frac{\text{Breaking stress}}{\text{Factor of safety}}$$

$$\text{Working stress} = \frac{480}{16} = 30 \text{ MN/m}^2$$

**Stress and Strain Units**

One Pascal is the pressure one Newton acting on an area of one square metre (1 Pa = 1 N/m<sup>2</sup>). The Newton and the Pascal are tiny units. As a result, force or pressure is usually referred to as kilo, mega or giga units. The table below summarises the abbreviations for multiple units of stress and strain.

Units	Description
N/m <sup>2</sup>	Newtons per Square Metre
KN/m <sup>2</sup>	KiloNewtons per Square Metre (Kilo = 10 <sup>3</sup> = 1000)
MN/m <sup>2</sup>	MegaNewtons per Square Metre (Mega = 10 <sup>6</sup> = 1 000 000)
GN/m <sup>2</sup>	GigaNewtons per Square Metre (Giga = 10 <sup>9</sup> = 1 000 000 000)
MPa	Mega Pascals (Mega = 10 <sup>6</sup> = 1 000 000)
GPa	Giga Pascals (Giga = 10 <sup>9</sup> = 1 000 000 000)

## Chapter 9: Refrigeration

Many substances meet the primary requirement of a refrigerant in that it can readily absorb heat and transfer it to another location and then give up again. In choosing a refrigerant, other factors must be considered including the refrigerant's stability and cost, the fire, explosive or poisoning risks, the ability to detect leaks, the corrosive effect on plant or equipment and the overall effect on the environment.

### Regulations and control

Some chlorine rich fluorocarbon refrigerants such as *chlorofluorocarbons* (CFCs) and *hydrochlorofluorocarbons* (HCFCs) cause severe environmental damage by depleting the earth's protective ozone layer. At the Montreal Convention in 1987, international action initiated banning or phasing out the worst of these substances. *Bromofluorocarbons* (Halon) and *hydrobromofluorocarbons* (HBFCs) are now banned and signatory countries control use by restrictions on sales and operating requirements (licensing) for repairers.

The Commonwealth is a party to the international agreements to further controls. Since 1996 CFCs have been banned and HCFCs have been classified as controlled substances requiring special licences. A total ban of HCFCs is scheduled by 2020. The control is devolved to State and Territory governments whose enforcement agencies include their Environmental Protection Agencies, Workcover and Maritime Authorities.

Industry associations and manufacturers are sources for product information regarding usage restrictions and handling in product *hazardous material data safety sheets*. Some existing refrigeration plants will not be economic to adapt to the use of a newer compliant refrigerants. Awareness of the currently banned or restricted use refrigerants is critical in order to monitor compliance of the system on your vessel. While the refrigerants are carefully controlled so are repairers and refrigerant sales- both activities require licensing.

### Refrigerants safety

When dealing with refrigerants special care must be taken to avoid both personal injury and environmental damage when filling or purging the system, or if a leak occurs. Detectors are available for refrigeration system leakage, the most common called a *halide leak detector*.

PPEs are advised while working on systems. All refrigerants on contact with the skin will cause a freeze burn. Due to its pressurisation, the possibility of squirts into the eyes with leaks is a high possibility. In the event of an accident immediate first aid will include irrigation and seeking medical assistance. Fluorocarbon refrigerants are

heavier than air and can lead to asphyxiation and death in confined spaces. In affected persons fresh air and resuscitation will be required immediately.

Refrigerant storage cylinders should be stowed with their thread caps fitted to prevent damage, kept away from the sun or heat and never be more than  $\frac{3}{4}$  full. Refrigerant storage cylinders are colour coded to Australian Standards for initial identification. The correct refrigerant and coloured cylinder *must* always be used appropriately. However, be wary that this precaution has carelessly not been observed.

To positively identify the refrigerant, a compound gauge can be fitted to measure the cylinder pressure and the temperature at the bottom of the cylinder. Using a *pressure temperature chart* the refrigerant can be identified.

## **Types of refrigerants**

Different refrigerants are used dependant on the refrigeration system and its purpose. They are grouped under three main types:

*Group One: Safest of the refrigerants*

*Group Two: Toxic and some flammable refrigerants*

*Group Three: Flammable refrigerants*

Despite some of these refrigerants being called non-toxic, care must be taken when handling the material to avoid any possibility personal injury. By itself a refrigerant may be relatively harmless, but having been introduced to the air, flame or hot metal, the refrigerant may break down into dangerous by-products such as acid or gas.

Listed below are common refrigerants, their characteristics and associated hazards:

### **Group One**

*R12 Dichlorodifluoromethane -is a popular liquid refrigerant which is colourless and almost odourless. It is not toxic, does not irritate, does not corrode and is non-flammable. Found in air conditioning (especially in vehicles) and refrigeration systems.*

*R22 Monochlorodifluoromethane - is a synthetic refrigerant and is used in refrigeration systems which a low evaporating temperature. It is not toxic, does not irritate, does not corrode and is non-flammable. Used in air conditioners and domestic and commercial refrigeration systems.*



*R134a Tetrafluoroethane - is the common replacement for many R12 applications where a medium evaporating temperature is required.*

*R502 Azeotropic mixture - This is a liquid mixture of refrigerants and is non-flammable, non-corrosive and is generally not toxic. R502 is used in systems requiring low to medium evaporating temperatures. Generally used in commercial refrigeration systems.*

## **Group Two**

*R717 Ammonia - is common in industrial applications and is a chemical compound. It is flammable and can form an explosive mixture with air. In small quantities R717 is not poisonous but will severely irritate the respiratory system. Large exposure must be avoided. The gas has a strong and ugly odour and is easily detected. Exposure can cause burns, eye injury and loss of consciousness. Protective clothing and breathing apparatus should be worn when dealing with this refrigerant.*

## **Group Three**

*Include R170 Ethane, R290 Propane, R600 Butane, R601 Isobutane and R1150 Ethylene.*

*All are highly flammable.*

## **Temperature pressure tables**

These tables are available from your refrigerant gas supplier and they show the pressure that a refrigerant requires to change phase - it indicates the saturation temperature of a number of common refrigerants at a given pressure and consequently can also be used to identify the type of refrigerant.

To determine the pressure for R22 to vapourise at -12°C, scan across from the tabulated temperature to the pressure reading of 229 kPa. To find the gauge pressure required from the absolute pressures tabulated, one atmosphere has to be subtracted:

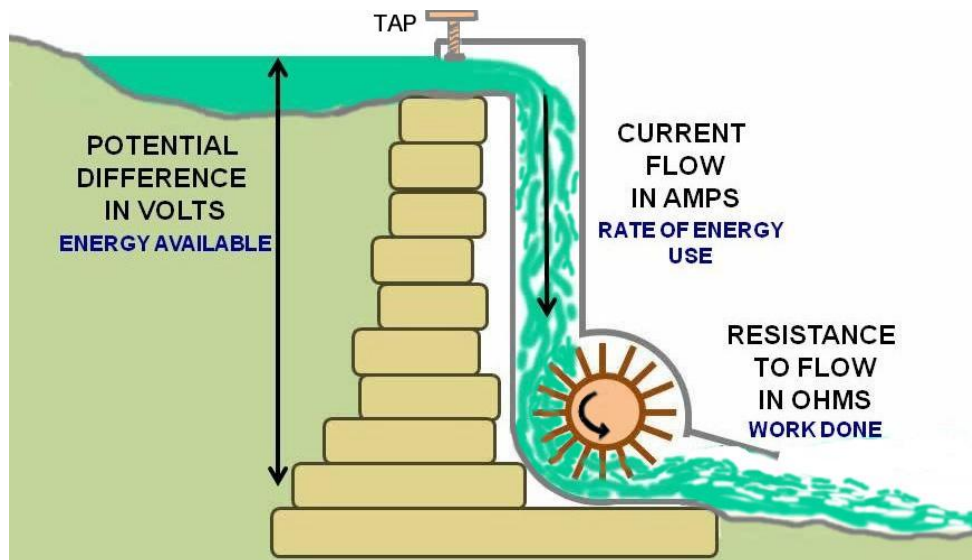
229 kPa (absolute pressure) -100 kPa (atmospheric pressure) = 119 kPa (gauge pressure)

<b>Refrigerant Pressure Temperature Chart</b> <i>For example and study use only courtesy of Reece plumbing supplies <a href="http://www.reece.com.au">http://www.reece.com.au</a></i>								
°C	R22	R134a	R507	R410A	R404A	R404A	R407C	R407C
	Saturated Conditions				Bubble	Dew	Bubble	Dew
	kPa	kPa	kPa	kPa	kPa	kPa	kPa	kPa
-40	4	-50	37	73	34	30	19	-16
-38	14	-45	50	90	47	42	30	-7
-36	25	-38	64	108	60	55	43	3
-34	37	-32	79	126	75	69	56	14
-32	49	-25	95	147	90	85	71	25
-30	63	-17	111	168	106	101	86	37
-28	77	-9	129	191	124	118	102	51
-26	92	0	148	215	143	137	119	65
-24	108	10	169	241	162	156	138	80
-22	126	20	190	269	183	177	158	96
-20	144	31	213	298	206	199	179	113
-18	163	43	237	329	229	222	201	132
-16	184	56	263	362	254	247	224	152
-14	206	69	290	396	281	273	249	172
-12	229	84	318	433	308	300	276	195
-10	253	99	348	471	338	329	303	218
-8	279	116	379	512	369	360	333	244
-6	306	133	413	555	401	392	364	270
-4	335	151	448	600	435	426	396	298
-2	365	171	484	647	471	462	430	328
0	397	191	523	697	509	499	467	359
2	430	213	563	749	548	538	504	392
4	465	236	605	804	590	579	544	427
6	501	261	649	861	633	622	586	464
8	540	286	696	921	678	667	629	503
10	580	313	744	983	726	714	675	544
12	621	342	794	1049	775	764	723	586
14	665	372	847	1118	827	815	773	631
16	711	403	902	1189	881	869	825	678
18	759	436	960	1264	937	925	879	727
20	809	470	1020	1342	996	983	936	779
22	861	507	1082	1423	1057	1044	995	833
24	915	544	1147	1507	1120	1107	1057	889
26	971	584	1214	1595	1187	1173	1121	949
28	1030	626	1284	1687	1255	1242	1188	1010
30	1091	669	1357	1782	1327	1313	1258	1075
32	1154	714	1433	1881	1401	1387	1330	1142
34	1220	761	1512	1984	1479	1464	1405	1212
36	1288	810	1594	2091	1559	1544	1483	1285
38	1359	862	1679	2202	1642	1627	1564	1361
40	1432	915	1767	2317	1728	1713	1648	1440

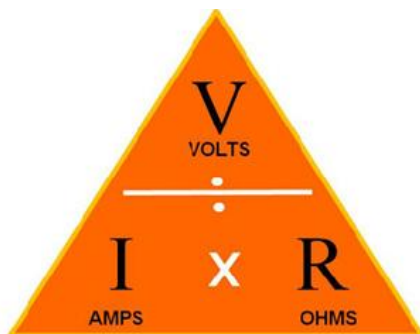
## Chapter 10: Electrical Calculations

We cannot see the electricity, but with appropriate electrical gauges we can measure a *potential difference* across the battery (in units called *volts*), a *current* along the cable (in *amperes*) and *resistance* to turn the motor (in *ohms*).

This relationship of these units can be compared with the drawing below. The top dam contains a pressure head of water (like voltage). If the tap is turned on the water flows down the pipe (like current through a conductor) and spins a water wheel below. In overcoming the water wheel's reluctance to rotate work is done and resistance is created in the water circuit.



The relationship of electrical pressure, current flow and resistance (called *Ohms Law*), is shown in the triangle below. If we know two quantities we can calculate the third and so monitor the performance of our system (its electrical *rating*).



$$\text{Volts (V)} = \text{A} \times \Omega$$

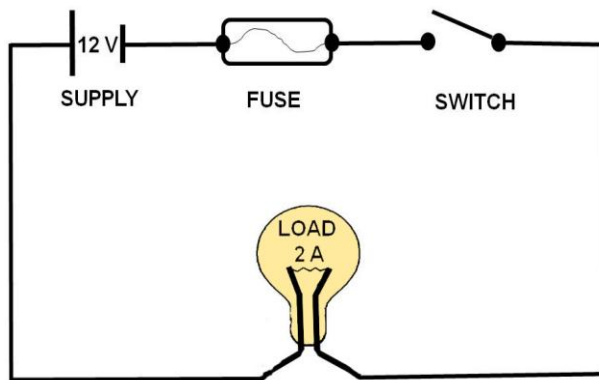
$$\text{Amps (A)} = \text{V} \div \Omega$$

$$\text{Ohms } (\Omega) = \text{V} \div \text{A}$$

## Simple circuits

In the next circuit there is a battery with a potential of 12 volts and a load (light globe) that when switched on will demand 2 amps to burn brightly. Conductors resist the flow of electric current (not super conductors) and convert a portion of the electrical energy into heat.

That work done will create a resistance to flow that we can calculate using *Ohms Law - The ratio of the potential difference (volts) between the ends of a conductor and the current flowing through the conductor is constant*. This ratio is called the resistance of the conductor and is expressed in the circuit and formula below:



$$\text{Volts} \div \text{Amps} = \text{Resistance}$$

$$12 \text{ V} \div 2 \text{ A} = 6 \Omega$$

$$\text{Volts} \times \text{Amps} = \text{Power}$$

$$12 \times 2 \text{ A} = 24 \text{ Watts}$$

Commonly the power consumption of a load is described by the unit called *watts*, where:

$$\text{Watts} = \text{Volts} \times \text{Amps}$$

In the circuit above it can be calculated that the globe rating required is 24 watts.

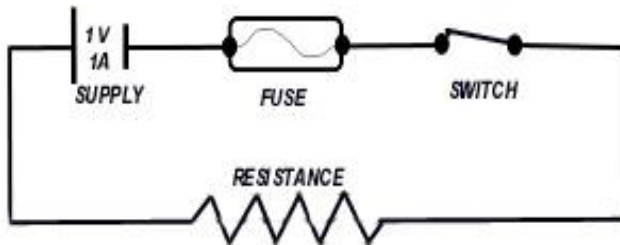
$$24 \text{ Watts} = 12 \text{ V} \times 2 \text{ A}$$

Watts (W) or Joules (P) are defined as units for the amount of work done in one second with a potential difference of one volt and a constant current of one ampere.

$$P = V \times A \quad P = A^2 \times \Omega \quad P = V^2 / \Omega$$

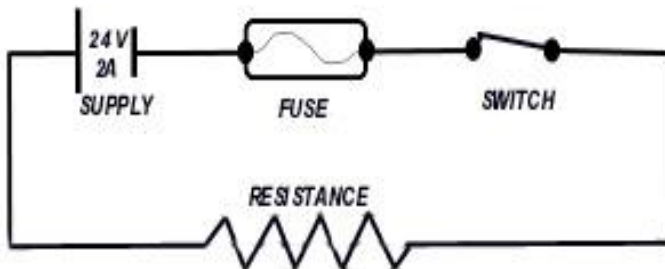
In the following diagrams:

a.)



$$R = \frac{V}{A} = \frac{1}{1} = 1 \Omega \quad P = V \times A = 1 \times 1 = 1 \text{ Watt}$$

b.)



$$R = \frac{V}{I} = \frac{24}{2} = 12 \Omega \quad P = V \times A = 24 \times 2 = 48 \text{ Watts}$$

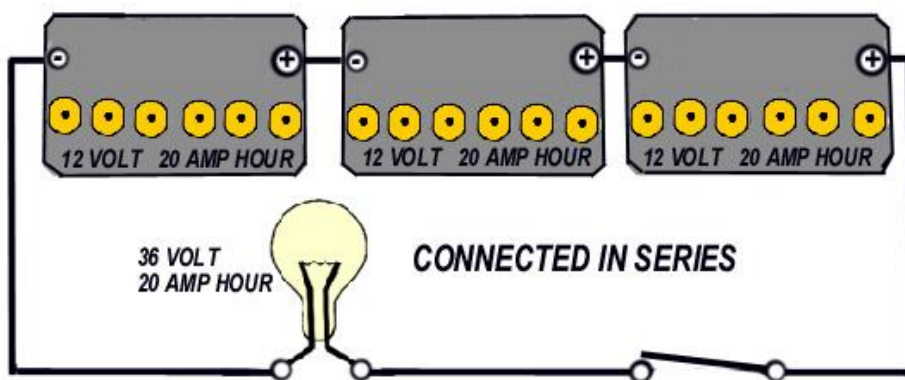
The circuit above includes a basic safety feature called a fuse. If the circuit is *overloaded* then the thin sacrificial wire of the fuse will glow red hot, melt and break before serious damage can occur to the circuit's cabling. This could happen if a *short circuit* occurred (where the cable's insulation is damaged and current bypasses the globe, typically by tracking through the metal of the vessel's hull or other exposed conductor). Clearly, keeping the insulation of our cables in tip top condition is a high

priority for electrical safety. Don't drag power cords over sharp surfaces, coil too tightly or expose to deterioration by the heat, weather or chemicals.

### Series and parallel circuits

Series and parallel connections are made when a large storage capacity is needed but a single battery would be too heavy or cumbersome to handle.

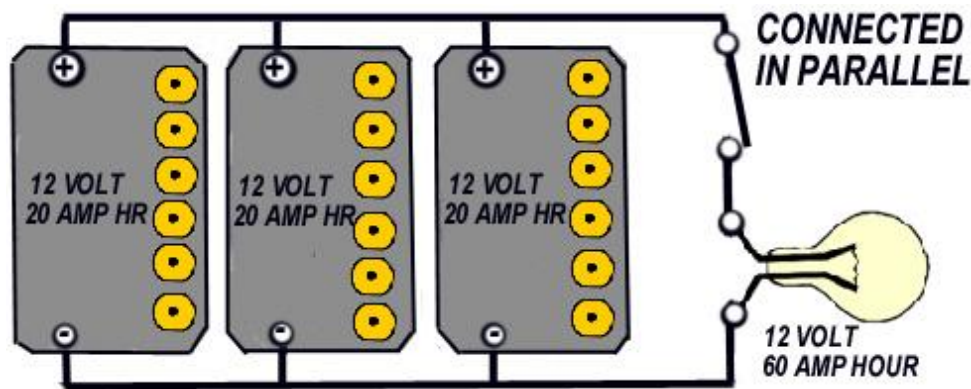
**Series Circuits-** If two or more batteries are connected in line with each other, they are said to be connected in series. The total voltage would be the sum of the individual batteries added together. While the output voltage increases, the total amp-hour capacity of the three batteries together remains the same as the amp-hour rating of one. In the case below the circuit provides 36 volts at 20 amps for an hour.



Similarly other components such as resistors (electrical devices that create resistance) connected in series would have their total resistance increased.

**Parallel Circuits** - If two or more batteries have all their positive poles linked and all their negative poles linked they are said to be connected in parallel. The total voltage would be the same as one of the batteries. While the output voltage remains the same, the total

amp-hour capacity of the three linked batteries increases. In the case below the circuit provides 12 volts at 60 amps for an hour.



The total power available would be:

$$\text{Volts} \times \text{Amps} = 12 \times 60 = 3600 \text{ watts.}$$

The resistance of the circuit would be equal to the sum of the resistance of each path.

From the example above, the Ohms Law formula can be converted to:


$$R = \frac{12V}{20A + 20A + 20A} = \frac{12V}{60A} = 0.2 \Omega$$

In the case of three batteries in parallel, they still have the same voltage, but there is an increase in the capacity. This means the linked batteries could be used for three times longer when charged but the battery with the highest terminal voltage will regulate the charger. This means the battery with the lower terminal voltage may never reach a fully charged condition.

### Battery testing of battery charge

The specific gravity of a lead acid batteries electrolyte is measured with a *hydrometer*. On battery discharge, the electrolyte becomes less dense, and the indicator in the hydrometer will sink. These batteries should be tested at least weekly. For safety, wait 30 minutes after charging/discharging and turn off all loads before taking measurements. Before removing the battery filler caps ensure adequate ventilation and remove naked flames. Insert the hydrometer it into each cell, squeezing its bulb to draw liquid into the chamber. Reading the scale at eye level will tell you the batteries condition. Typical readings are as follows:

<b>Charge</b>	<b>S.G.</b>	<b>Voltage</b>
100%	1.26	12.75
80%	1.22	12.55
60%	1.19	12.35
40%	1.16	12.1
20%	1.13	11.85
0%	1.1	11.65



**A HYDROMETER**

### Alternator power ratings

The factors in rating an alternator are:

*Frequency and speed*

*Kilovolt ampere output in a 3 phase circuit*

*Number of phases and terminal voltage*

**Frequency and speed** - frequency varies with the speed and is controlled by the governor. It is calculated by:

$$\text{Frequency (Hertz)} = \frac{\text{Speed} \times \text{Number of Poles}}{120}$$

**120**

120° is a constant, being 1/3 of the 360° electrical degrees of the three phase wave form. Most alternators are 4 pole machines.

*Example* - What speed must be achieved to develop a frequency of 50 Hertz using a 4 pole alternator:

$$\text{Frequency (Hertz)} = \frac{\text{Speed} \times \text{Number of Poles}}{120}$$

120

$$\text{Speed} = \frac{\text{Frequency} \times 120}{\text{Number of Poles}} = \frac{50 \times 120}{4} = 1500 \text{ rpm}$$



**Phases and terminal voltage-** Australian vessels commonly use 3 phase 415 volt. Larger motors are designed to run at these values. Lower voltage rated equipment of 240V, 115V, 24V use supply stepped down by transformers.

Alternators can be described by their Kilovolt Ampere output. In a 3 phase circuit this is calculated by:

$$1 \text{ kilovolt ampere (KVA)} = \frac{\sqrt{3} \times \text{Voltage (V)} \times \text{Current (I)}}{1000}$$

*Example* - How much current could a 20 KVA machine deliver at 415V?

$$\text{KVA} = \frac{\sqrt{3} \times \text{Voltage (V)} \times \text{Current (I)}}{1000}$$

$$I = \frac{1000 \times \text{KVA}}{\sqrt{3} \times V} = \frac{1000 \times 20}{\sqrt{3} \times 415} = 27.82 \text{ amps/phase.}$$

Sometimes alternators are rated in Kilo Watts (KW). A value called the *power factor* (cosine Ø) is introduced to moderate the previous formula.

$$\text{kW} = \frac{\sqrt{3} \times V \times I \times \cos \varnothing}{1000}$$

Simply put, the *power factor* is the ratio of True Power (kW) to Apparent Power (KVA).

Resistive loads such as incandescent lights and heaters have a power factor of 1.

Inductive loads such as motors and transformers have a power factor of approx. 0.8.

Sometimes when the alternator is rated in KW, a power factor, usually 0.8, is specified. The power factor can be used to calculate unknown values in the formula:

*Example* - Calculate the current from a 20 kW 415V alternator with a power factor of 0.8:

$$kW = \frac{\sqrt{3} \times V \times I \times \cos \phi}{1000}$$

$$I = \frac{1000kW}{\sqrt{3} \times V \cos \phi} = \frac{1000 \times 20}{\sqrt{3} \times 415 \times 0.8} = 34.8 \text{ amps/phase}$$

## Acknowledgements

Bulmershe Teachers Training College Maths Dept, Reading, Berkshire, UK  
 ANTA and Tafe Publications  
 Wikipedia.com  
 Bahram Abedi

*Ranger Hope © 2014*